III. *Researches in Physical Astronomy.* By JOHN WILLIAM LUBBOCK, *Esq., V.P. and Treas. R.S.*

Read December 9, 1830.

IN last April I had the honour of presenting to the Society a paper containing expressions for the variations of the elliptic constants in the theory of the motions of the planets. The stability of the solar system is established by means of these expressions, if the planets move in a space absolutely devoid of any resistance *, for it results from their form that however far the approximation be carried, the eccentricity, the major axis, and the tangent of the inclination of the orbit to a fixed plane, contain only periodic inequalities, each of the three other constants, namely, the longitude of the node, the longi-

* When the body moves in a medium which resists according to any power of the velocity, the contrary obtains, the major axis and eccentricity acquiring a term which varies with the time, while the longitude of the perihelion and longitude of the epoch have only periodic inequalities. This results from the equations given in the former part of this paper, Phil. Trans. Part II. 1830, page 340.

$$\mathrm{d}\,a = -\,2\,c\,a\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n+1}{2}}\frac{(1-e\cos v)}{n}\,\mathrm{d}\,v$$

$$\mathrm{d}\,e = -\,2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\left(\frac{1-e^2}{2}\right)\cos\mathrm{d}\,v$$

$$e\,\mathrm{d}\,\varpi = -\,2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\frac{\sqrt{1-e^2}}{n}\sin v\,\mathrm{d}\,v$$

$$\mathrm{d}\,\varepsilon - \mathrm{d}\,\varpi = 2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\left\{\frac{1-e^3\cos v}{e}\right\}\mathrm{d}\,v$$

$$\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n+1}{2}} = 1 + (n+1)\,e\cos v + (n+1)^2\,e^2\cos^2 v$$

$$\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}} = 1 + (n-1)\,e\cos v + (n-1)^2\,e^2\cos^2 v$$

tude of the perihelion, and the longitude of the epoch, contains a term which varies with the time, and hence the line of apsides and the line of nodes revolve continually in space. The stability of the system may therefore be inferred, which would not be the case if the eccentricity, the major axis, or the tangent of the inclination of the orbit to a fixed plane contained a term varying with the time, however slowly.

The problem of the precession of the equinoxes admits of a similar solution; of the six constants which determine the position of the revolving body, and the axis of instantaneous rotation at any moment, three have only periodic inequalities, while each of the other three has a term which varies with the time. From the manner in which these constants enter into the results, the equilibrium of the system may be inferred to be stable, as in the former case. Of the constants in the latter problem, the mean angular velocity of rotation

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$$\mathrm{d}\,a = -\frac{2ca}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \{1 + n e \cos v + n(n+1)\, e^2 \cos^2 v + \&c.\}\, \mathrm{d}\, v$$

$$= -\frac{2ca}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \left\{1 + \frac{n(n+1)}{2} e^2 + n e \cos v + \frac{n(n+1)}{2} e^2 \cos^2 v + \&c.\right\} \mathrm{d}\, v$$

$$\mathrm{d}\,e = -\frac{2c}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \{\cos v + (n+1)\, e \cos^2 v + (n+1)^2 e^2 \cos^3 v + \&c.\}\,(1 - e^2)\, \mathrm{d}\, v$$

$$= -\frac{2c}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \left\{\frac{n+1}{2} e + \left(1 + 3\,\frac{(n+1)^2}{4}\right) \cos v + \frac{(n+1)}{2}\, e \cos 2\, v \right.$$

$$\left. + \frac{(n+1)^2}{4} \cos 3\, v + \&c.\right\} (1 - e^2)\, \mathrm{d}\, v$$

neglecting the terms which are periodic

$$\mathrm{d}\,a = -\frac{2ca}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \left\{1 + \frac{n(n+1)}{2} e^2 + \&c.\right\} \mathrm{d}\, v$$

$$\mathrm{d}\,e = -\frac{2c}{n}\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}} \left\{\frac{(n+1)}{2} e + \&c.\right\} (1 - e^2)\, \mathrm{d}\, v$$

The major axis decreases perpetually, the eccentricity diminishes perpetually until it reaches zero, while the perihelion retains the same *mean position*, and the longitude of the epoch the same *mean value*. I stated inadvertently in the former part of this paper, p. 340, that the variations of the eccentricity are all periodical.

may be considered analogous to the mean motion of a planet, or its major axis; the geographical longitude, and the cosine of the geographical latitude of the pole of the axis of instantaneous rotation, to the longitude of the perihelion and the eccentricity; the longitude of the first point of Aries and the obliquity of the ecliptic, to the longitude of the node and the inclination of the orbit to a fixed plane; and the longitude of a given line in the body revolving, passing through its centre of gravity, to the longitude of the epoch. By the stability of the system I mean that the pole of the axis of rotation has always nearly the same geographical latitude, and that the angular velocity of rotation, and the obliquity of the ecliptic vary within small limits, and periodically. These questions are considered in the paper I now have the honour of submitting to the Society. It remains to investigate the effect which is produced by the action of a resisting medium; in this case the latitude of the pole of the axis of rotation, the obliquity of the ecliptic, and the angular velocity of rotation might vary considerably, although slowly, and the climates undergo a considerable change.

The co-efficients of the terms in the development of R, multiplied by the squares and products of the eccentricities, are susceptible of very great simplification, in consequence of the equations of condition which obtain between the quantities of which the general symbol is b. I have now given the development of R, as far as the terms depending upon the squares and products of the eccentricities, in its simplest form. See p. 30.

I have also given methods of obtaining the inequalities of the radius vector, of longitude, and of latitude in the planetary theory. The expressions in this paper differ in form from those of LAPLACE, but their identity may be shown by means of equations of condition which obtain between some of the quantities involved.

I have taken as a numerical example, the calculation of the co-efficients of some of the inequalities in the theory of Jupiter, disturbed by Saturn.

On the Precession of the Equinoxes.

Let O be the origin of the co-ordinate axes, coinciding with some point in the interior of the mass M.

Let x, y, z be the co-ordinates of any element dm parallel to three rectangular axes O x, O y, O z, fixed in space, $x_{,}$, $y_{,}$, $z_{,}$, the co-ordinates of the same element parallel to three other rectangular axes O $x_{,}$, O $y_{,}$, O $z_{,}$, fixed in the mass M' and revolving with it. Let the line N O N$'$ be the intersection of the plane $x_{,} y_{,}$ with the plane $x y$,

Let the angle N O $x = \psi$, N O $x_{,} = \varphi$, and the inclination of the plane $x_{,} y_{,}$ upon $x y = \theta$.

$$x_{,} = x(\cos\theta \sin\psi \sin\varphi + \cos\psi \cos\varphi) + y(\cos\theta \cos\psi \sin\varphi - \sin\psi \cos\varphi) - z \sin\theta \sin\varphi$$
$$y_{,} = x(\cos\theta \sin\psi \cos\phi - \cos\psi \sin\phi) + y(\cos\theta \cos\psi \cos\varphi + \sin\psi \sin\varphi) - z \sin\theta \cos\varphi$$
$$z_{,} = x \sin\theta \sin\psi + y \sin\theta \cos\psi + z \cos\theta$$

Let $X_{,}$, $Y_{,}$, $Z_{,}$ be the accelerating forces which act upon the element dm in the direction of the axes O $x_{,}$, O $y_{,}$, O $z_{,}$,

$$\int (y_{,}^2 + z_{,}^2)\, d m = A, \qquad \int (x_{,}^2 + z_{,}^2)\, d m = B, \qquad \int (x_{,}^2 + y_{,}^2)\, d m = C$$

$$p\, \mathrm{d} t = \sin\varphi \sin\theta\, \mathrm{d}\psi - \cos\varphi\, \mathrm{d}\theta$$
$$q\, \mathrm{d} t = \cos\varphi \sin\theta\, \mathrm{d}\psi + \sin\varphi\, \mathrm{d}\theta$$
$$r\, \mathrm{d} t = \mathrm{d}\varphi - \cos\theta\, \mathrm{d}\psi$$

$$C \mathrm{d} r + (B - A) p\, q\, \mathrm{d} t = \mathrm{d} t \int (x_{,} Y_{,} - y_{,} X_{,})\, \mathrm{d} m$$

$$B \mathrm{d} q + (A - C) r\, p\, \mathrm{d} t = \mathrm{d} t \int (z_{,} X_{,} - x_{,} Z_{,})\, \mathrm{d} m$$

$$A \mathrm{d} p + (C - B) q\, r\, \mathrm{d} t = \mathrm{d} t \int (y_{,} Z_{,} - x_{,} Y_{,})\, \mathrm{d} m$$

If the axis of instantaneous rotation coincides with the line O I at any instant

$$\cos \mathrm{I\,O}\, x_{,} = \frac{p}{\sqrt{p^2 + q^2 + r^2}}$$

$$\cos \mathrm{I\,O}\, y_{,} = \frac{q}{\sqrt{p^2 + q^2 + r^2}}$$

$$\cos \mathrm{I\,O}\, z_{,} = \frac{r}{\sqrt{p^2 + q^2 + r^2}}$$

$$\sin \mathrm{I\,O}\, z_{,} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + r^2}}$$

If $z_{,}$ I L be a great circle cutting the plane $x_{,} y_{,}$ in L,

$$\cos x_{,} \mathrm{O\,L} = \frac{\cos \mathrm{I\,O}\, x_{,}}{\sin \mathrm{I\,O}\, z_{,}} = \frac{p}{\sqrt{p^2 + q^2}}$$

If the accelerating forces $X, Y, Z = 0$ and $B = A$, the integrals of the preceding equations are

$$r = n, \qquad p = c \cos \frac{C - A}{A}(n\, t + \nu), \qquad q = c \sin \frac{C - A}{A}(n\, t + \gamma)$$

and neglecting c^2,

$$\theta = \omega + \frac{c}{n}\frac{A}{C}\sin\left(\frac{C}{A}\,n\,t + \frac{C-A}{A}\gamma\right)$$

$$\psi = \psi_0 - \frac{c}{n\sin\omega}\frac{A}{C}\cos\left(\frac{C}{A}\,n\,t + \frac{C-A}{A}\gamma\right)$$

$$\varphi = \varphi_0 + n\,t - \frac{c\cos\omega}{n\sin\omega}\frac{A}{C}\cos\left(\frac{C}{A}\,n\,t + \frac{C-A}{A}\gamma\right)$$

n, c, γ, ω, ψ_0, and φ_0 being constants. In the problem of the Precession of the Equinoxes ω is the mean *obliquity of the ecliptic*, ψ_0 is the *longitude of the first point of Aries* when $t = 0$ reckoned from some fixed line.

Sin I O $z_, = \frac{c}{\sqrt{n^2 + c^2}}$, hence it appears that if a body whose form is that of a figure of revolution be made to revolve, and be acted upon by no extraneous force, the axis of instantaneous rotation revolves about the axis of the figure, the *latitude* of the former axis remaining constant.

$$\text{The angle } x_, \text{O I}_, = \frac{C-A}{A}(n\,t + \gamma)$$

If the forces $X_,$, $Y_,$, $Z_,$ arise from the attractions of a distant luminary M', of which the coordinates referred to the axes O $x_,$, O $y_,$, O $z_,$ are $x'_,$, $y'_,$, $z'_,$, the force varying inversely as the square of the distance,

$$X_, = \frac{M'(x_,' - x_,)}{\{(x_, - x_,')^2 + (y_, - y_,')^2 + (z_, - z_,')^2\}^{\frac{3}{2}}}$$

$$Y_, = \frac{M'(y_,' - y_,)}{\{(x_, - x_,')^2 + (y_, - y_,')^2 + (z_, - z_,')^2\}^{\frac{3}{2}}}$$

$$Z_, = \frac{M'(z_,' - z_,)}{\{(x_, - x_,')^2 + (y_, - y_,')^2 + (z_, - z_,')^2\}^{\frac{3}{2}}}$$

$$\int(x_, Y_, - y_, X_,)\,\mathrm{d}m = M'\int\frac{(x_, y_,' - y_, x_,')}{r'^3}\left\{1 + \frac{3(x_, x_,' + y_, y_,' + z_, z_,')}{r^2} + \&\text{c.}\right\}\mathrm{d}m$$

By the properties of the principal axes $\int x_, y_,\,\mathrm{d}m = 0$, $\int x_, z_,\,\mathrm{d}m = 0$, $\int y_, z_,\,\mathrm{d}m = 0$, and by the properties of the centre of gravity $\int x_,\,\mathrm{d}m = 0$, $\int y_,\,\mathrm{d}m = 0$, $\int z_,\,\mathrm{d}m = 0$, whence

$$\int(x_, Y_, - y_, X_,)\,\mathrm{d}m = \frac{3M' x_,' y_,'}{r'^5}\int(x_,^2 - y_,^2)\,\mathrm{d}m = \frac{3M'(B-A)}{r'^5}x_,' y_,'\,\mathrm{d}t$$

$$C\,\mathrm{d}r + (B - A)pq\,\mathrm{d}t = \frac{3\,M'\,(B - A)}{r'^5}\,x_{,}\,y_{,}'\,\mathrm{d}t$$

$$B\,\mathrm{d}q + (A - C)rp\,\mathrm{d}t = \frac{3\,M'(A - C)}{r'^5}\,z_{,}'\,x_{,}'\,\mathrm{d}t$$

$$A\,\mathrm{d}p + (C - B)qr\,\mathrm{d}t = \frac{3\,M'(C - B)}{r'^5}\,y_{,}'\,z_{,}'\,\mathrm{d}t$$

Substituting for $x_{,}'$, $y_{,}'$, $z_{,}'$ their values from equations, p. 20, upon the supposition of $\psi = 0$,

$$C\,\mathrm{d}r + (B - A)pq\,\mathrm{d}t = \frac{3\,M'(B - A)}{2\,r'^5}\left\{\{(y'\cos\theta - z'\sin\theta)^2 - x'^2\}\sin 2\varphi \right.$$
$$\left. + 2x'(y'\cos\theta - z'\sin\theta)\cos^2\varphi\right\}$$

$$B\,\mathrm{d}q + (A - C)rp\,\mathrm{d}t = \frac{3\,M'(A - C)}{r_{,}^5}\left\{x'(y'\sin\theta + z'\cos\theta)\cos\varphi \right.$$
$$\left. + (y'\cos\theta - z'\sin\theta)(y'\sin\theta + z'\cos\theta)\sin\varphi\right\}$$

$$A\,\mathrm{d}p + (C - B)qr\,\mathrm{d}t = \frac{3\,M'(C - B)}{r'^5}\left\{(y'\cos\theta - z'\sin\theta)(y'\sin\theta + z'\cos\theta)\cos\varphi \right.$$
$$\left. - x'(y'\sin\theta + z'\cos\theta')\sin\varphi\right\}$$

If

$$\frac{3\,M'}{r'^5}\left\{(y'\cos\theta - z'\sin\theta)(y'\sin\theta + z'\cos\theta)\right\} = P$$

$$\frac{3\,M'}{r'^5}\left\{x'(y'\sin\theta + z'\cos\theta)\right\} = P'$$

$$B\,\mathrm{d}q + (A - C)rp\,\mathrm{d}t = (A - C)\,\mathrm{d}t\,\{P'\cos\varphi + P\sin\varphi\}$$

$$A\,\mathrm{d}p + (C - B)qr\,\mathrm{d}t = (C - B)\,\mathrm{d}t\,\{P\cos\varphi - P'\sin\varphi\}$$

P and P' may be developed according to sines and cosines of angles increasing proportionally to the time. Let $k\cos(it + \varepsilon)$ be any term of P, $k'\sin(it + \varepsilon)$ the corresponding term of P',

$$B\,\mathrm{d}q + (A - C)rp\,\mathrm{d}t = \frac{A - C}{2}\,\mathrm{d}t\left\{(k + k')\sin(\phi + it + \varepsilon) + (k - k')\sin(\varphi - it - \varepsilon)\right\}$$

$$A\,\mathrm{d}p + (C - B)qr\,\mathrm{d}t = \frac{C - B}{2}\,\mathrm{d}t\left\{(k + k')\cos(\varphi + it + \varepsilon) + (k - k')\cos(\varphi - it - \varepsilon)\right\}$$

The equations which were given p. 21, may still be considered as affording a solution of the problem by making the constants n, c, γ, ω, ψ_0 and φ_0 vary,

$$\mathrm{d}n = 0$$

$$\sin\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}c + c\,\frac{C-A}{A}\cos\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}\gamma + \frac{(A-C)}{A}\,nc\cos\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}t$$
$$= \frac{A-C}{2A}\,\mathrm{d}t\left\{(k+k')\sin(\phi+it+\varepsilon) + (k-k')\sin(\varphi-it-\varepsilon)\right\}$$

$$\cos\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}c - c\,\frac{C-A}{A}\sin\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}\gamma - \frac{A-C}{A}\,nc\sin\frac{C-A}{A}(nt+\gamma)\,\mathrm{d}t$$
$$= -\frac{A-C}{2A}\,\mathrm{d}t\left\{(k+k')\cos(\varphi+it+\varepsilon) + (k-k')\cos(\varphi-it-\varepsilon)\right\}$$

since $\varphi = \varphi_0 + nt$ nearly

$$\mathrm{d}c = \frac{A-C}{2A}\,\mathrm{d}t\left\{-(k+k')\cos\left(\varphi_0+nt+\frac{C-A}{A}(nt+\gamma)+it+\varepsilon\right)\right.$$
$$\left.-(k-k')\cos\left(\varphi_0+nt+\frac{C-A}{A}(nt+\gamma)-it-\varepsilon\right)\right\}$$

$$c\,\frac{C-A}{A}\,\mathrm{d}\gamma + \frac{A-C}{A}\,nc\,\mathrm{d}t = \frac{A-C}{2A}\,\mathrm{d}t\left\{(k+k')\sin\left(\varphi_0+nt+\frac{C-A}{A}(nt+\gamma)+it+\varepsilon\right)\right.$$
$$\left.+(k-k')\sin\left(\varphi_0+nt+\frac{C-A}{A}(nt+\gamma)-it-\varepsilon\right)\right\}$$

$$\mathrm{d}\omega + \frac{\mathrm{d}c}{n}\frac{A}{C}\sin\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right) + \frac{c}{n}\,\frac{C-A}{C}\cos\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right)\mathrm{d}\gamma = 0$$

$$\mathrm{d}\psi_0 - \frac{\mathrm{d}c}{n\sin\omega}\,\frac{A}{C}\cos\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right) + \frac{c\cos\omega}{n\sin\omega^2}\,\frac{A}{C}\cos\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right)\mathrm{d}\omega$$
$$+\frac{c}{n\sin\omega}\,\frac{C-A}{A}\sin\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right)\mathrm{d}\gamma = 0$$

$$\mathrm{d}\varphi_0 - \frac{\mathrm{d}c}{n\sin\omega}\,\frac{A}{C}\cos\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right) + \frac{c}{n\sin\omega^2}\,\frac{A}{C}\cos\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right)\mathrm{d}\omega$$
$$+\frac{c}{n\sin\omega}\,\frac{C-A}{A}\sin\left(\frac{C}{A}nt+\frac{C-A}{A}\gamma\right)\mathrm{d}\gamma = 0$$

From the preceding expressions it may be inferred that

$n =$ constant.

$\frac{\mathrm{d}c}{\mathrm{d}t} =$ series of cosines without any constant quantity, unless the mean motion of rotation is commensurate to the mean motion of revolution of the luminary M'.

$\frac{d\gamma}{dt}$ = series of sines without any constant quantity, except in a similar case.

c being equal to a constant + a series of sines.

$\frac{d\omega}{dt}$ = a series of sines without any constant quantity.

$\frac{d\psi_0}{dt}$ = a series of cosines + a constant quantity.

$\frac{d\varphi_0}{dt}$ = a series of cosines + a constant quantity.

In the general case where A is not equal to B, n = constant + series of cosines.

The form of the preceding expressions is not affected however, for the approximation may be carried so that except in the case of commensurability above mentioned, the mean motion of rotation being also nearly twice the mean motion of the planet M' in its orbit or greater,

n = constant + series of cosines without any constant quantity multiplied by the time.

c = constant + series of sines or cosines without any constant quantity multiplied by the time.

ω = constant + series of cosines without any constant quantity multiplied by the time.

γ = constant + series of cosines or sines + a constant quantity multiplied by the time.

ψ_0 = constant + series of sines + a constant quantity multiplied by the time.

φ_0 = constant + series of sines + a constant quantity multiplied by the time.

The constant quantity multiplied by the time in the value of ψ_0 is the precession of the equinox.

If $z' = 0$, (which amounts to taking for the fixed plane the orbit of the planet M',) and n' be its mean motion, then neglecting the eccentricity, $x' = a' \cos n' t$, $y' = a' \sin n' t$, $r' = a'$,

$$P = \frac{3\,M'}{2\,r'^3} \sin \omega \cos \omega \,(1 - \cos 2\,n'\,t), \qquad P' = \frac{3\,M'}{2\,r'^3} \sin \omega \sin 2\,n'\,t$$

Supposing $\varphi_0 = 0$, $\gamma = 0$, $c_0 = 0$, c_0 being in fact imperceptible to observation, and neglecting $\cos 2n't$, $\sin 2n't$, in order to find the constant part of $\frac{d\psi_0}{dt}$,

$$\frac{d c_0}{d t} = -\frac{3(A-C)}{2nAa'^3} M' \sin\omega\cos\omega\cos\frac{C}{A}n't$$

$$\frac{c\,d\gamma}{d t} = -\frac{3(A-C)}{2nAa'^3} M' \sin\omega\cos\omega\sin\frac{C}{A}n't$$

$$\frac{d\psi_0}{d t} = \frac{3(C-A)}{2nCa'^3} M'\cos\omega = \frac{3(C-A)}{2nC} n'^2\cos\omega$$

This result agrees with that given in the Méc. Cél. vol. ii. p. 318, and with that given by M. Poisson, Mémoires de l'Academie, vol. vii. p. 247. In Laplace's notation $\omega = h$, $m = n'$. In M. Poisson's notation $\omega = \theta$, $m = n'$.

On the Theory of the Motion of the Planets, continued from Part II. 1830, *p.* 357.

From the general equations given in the Méc. Cél. vol. i. p. 268, the following may be inferred.

$$\frac{3}{2}\frac{a}{a_{\prime}}b_{1,2} = \frac{(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{1,1} - \frac{a}{a_{\prime}}b_{1,0} \qquad \frac{5}{2}\frac{a}{a_{\prime}}b_{1,3} = \frac{2(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{1,2} - \frac{3}{2}\frac{a}{a_{\prime}}b_{1,1}$$

$$\frac{7}{2}\frac{a}{a_{\prime}}b_{1,4} = \frac{3(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{1,3} - \frac{5}{2}\frac{a}{a_{\prime}}b_{1,2} \qquad \frac{9}{2}\frac{a}{a_{\prime}}b_{1,5} = \frac{4(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{1,4} - \frac{7}{2}\frac{a}{a_{\prime}}b_{1,3}$$

$$\frac{a}{2a_{\prime}}b_{3,2} = \frac{(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{3,1} - \frac{3a}{a_{\prime}}b_{3,0} \qquad \frac{3}{2}\frac{a}{a_{\prime}}b_{3,3} = \frac{2(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{3,2} - \frac{5}{2}\frac{a}{a_{\prime}}b_{3,1}$$

$$\frac{5}{2}\frac{a}{a_{\prime}}b_{3,4} = \frac{3(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{3,3} - \frac{7}{2}\frac{a}{a_{\prime}}b_{3,2} \qquad \frac{7}{2}\frac{a}{a_{\prime}}b_{3,5} = \frac{4(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{3,4} - \frac{9}{2}\frac{a}{a_{\prime}}b_{3,3}$$

$$-\frac{a}{2a_{\prime}}b_{5,2} = \frac{(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{5,1} - \frac{5a}{a_{\prime}}b_{5,0} \qquad \frac{a}{2a_{\prime}}b_{5,3} = \frac{2(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{5,2} - \frac{7}{2}\frac{a}{a_{\prime}}b_{5,1}$$

$$\frac{3}{2}\frac{a}{a_{\prime}}b_{5,4} = \frac{3(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{5,3} - \frac{9}{2}\frac{a}{a_{\prime}}b_{5,2}' \qquad \frac{5}{2a_{\prime}}b_{5,5} = \frac{4(a^2+a_{\prime}^2)}{a_{\prime}^2}b_{5,4} - \frac{11}{2}\frac{a}{a_{\prime}}b_{5,3}$$

$$b_{1,0} = \frac{a^2+a_{\prime}^2}{a_{\prime}^2}b_{3,0} - \frac{a}{a_{\prime}}b_{3,1} \qquad b_{1,1} = \frac{a^2+a_{\prime}^2}{a_{\prime}^2}b_{3,1} - \frac{2a}{a_{\prime}}b_{3,0} - \frac{a}{a_{\prime}}b_{3}$$

$$b_{1,2}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{3,2}-\frac{a}{a_{,}}b_{3,1}-\frac{a}{a_{,}}b_{3,3} \qquad b_{1,3}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{3,3}-\frac{a}{a_{,}}b_{3,2}-\frac{a}{a_{,}}b_{3,4}$$

$$b_{3,0}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{5,0}-\frac{a}{a_{,}}b_{5,1} \qquad b_{3,1}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{5,1}-\frac{2a}{a_{,}}b_{5,0}-\frac{a}{a_{,}}b_{5,2}$$

$$b_{3,2}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{5,2}-\frac{a}{a_{,}}b_{5,1}-\frac{a}{a_{,}}b_{5,3} \qquad b_{3,3}=\frac{a^2+a_{,}^2}{a_{,}^2}b_{5,3}-\frac{a}{a_{,}}b_{5,2}-\frac{a}{a_{,}}b_{5,4}$$

$$b_{1,1}=\frac{a}{a_{,}}\left\{b_{3,0}-\tfrac{1}{2}b_{3,2}\right\} \qquad 2\,b_{1,2}=\frac{a}{2\,a_{,}}\left\{b_{3,1}-b_{3,3}\right\}$$

$$3\,b_{1,3}=\frac{a}{2\,a_{,}}\left\{b_{3,2}-b_{3,4}\right\} \qquad 4\,b_{1,4}=\frac{a}{2\,a_{,}}\left\{b_{3,2}-b_{3,4}\right\}$$

$$b_{3,1}=3\frac{a}{a_{,}}\left\{b_{5,0}-\tfrac{1}{2}b_{5,2}\right\} \qquad 2\,b_{3,2}=\frac{3}{2}\frac{a}{a_{,}}\left\{b_{5,1}-b_{5,3}\right\}$$

$$3\,b_{3,3}=\frac{3}{2}\frac{a}{a_{,}}\left\{b_{5,2}-b_{5,4}\right\} \qquad 4\,b_{3,4}=\frac{3}{2}\frac{a}{a_{,}}\left\{b_{5,3}-b_{5,5}\right\}$$

$$2\,b_{3,0}=\frac{2\left(1+\frac{a^2}{a_{,}^2}\right)b_{1,0}-2\frac{a}{a_{,}}b_{1,1}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}, \qquad b_{3,1}=\frac{3\left(1+\frac{a^2}{a_{,}^2}\right)b_{1,1}-2\,.\,3\frac{a}{a_{,}}b_{1,2}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}$$

$$b_{3,2}=\frac{5\left(1+\frac{a^2}{a_{,}^2}\right)b_{1,2}-2\,.\,5\frac{a}{a_{,}}b_{1,3}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}, \qquad b_{3,3}=\frac{7\left(1+\frac{a^2}{a_{,}^2}\right)b_{1,3}-2\,.\,7\frac{a}{a_{,}}b_{1,4}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}$$

$$2\,b_{5,0}=\frac{2\left(1+\frac{a^2}{a_{,}^2}\right)b_{3,0}+\frac{2}{3}\frac{a}{a_{,}}b_{3,1}}{\left(1-\frac{a^2}{a_{,}}\right)^2}, \qquad b_{5,1}=\frac{\frac{5}{3}\left(1+\frac{a^2}{a_{,}^2}\right)b_{3,1}-\frac{2}{3}\frac{a}{a_{,}}b_{3,2}}{\left(1-\frac{a^2}{a^2_{,}}\right)^2}$$

$$\tfrac{1}{2}b_{5,2}=\frac{\frac{7}{3}\left(1+\frac{a^2}{a_{,}^2}\right)b_{3,1}-2\frac{a}{a_{,}}b_{3,3}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}, \qquad b_{5,3}=\frac{\frac{9}{3}\left(1+\frac{a^2}{a_{,}^2}\right)b_{3,3}-\frac{2\,.\,5}{3}\frac{a}{a_{,}}b_{3,4}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}$$

$$2\,b_{7,0}=\frac{2\left(1+\frac{a^2}{a_{,}^2}\right)b_{5,0}+\frac{2\,.\,3}{5}\frac{a}{a_{,}}b_{5,1}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}, \qquad b_{7,1}=\frac{\frac{7}{5}\left(1+\frac{a^2}{a_{,}^2}\right)b_{5,1}+\frac{2}{5}\frac{a}{a_{,}}b_{5,2}}{\left(1-\frac{a^2}{a_{,}^2}\right)^2}$$

$$b_{7,2} = \frac{\frac{9}{5}\left(1 + \frac{a^2}{a_i^2}\right) b_{5,2} - \frac{2}{5}\frac{a}{a_i} b_{5,3}}{\left(1 - \frac{a^2}{a_i^2}\right)^2}, \qquad b_{7,3} = \frac{\frac{11}{5}\left(1 + \frac{a^2}{a_i^2}\right) b_{5,3} - \frac{2 \, . \, 3}{5}\frac{a}{a_i} b_{5,4}}{\left(1 - \frac{a^2}{a_i^2}\right)^2}$$

$$r = a F(n t), \qquad \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = \frac{\mathrm{d} R \, \mathrm{d} a}{\mathrm{d} a \, \mathrm{d} a F(n t)}, \qquad \frac{\mathrm{d} a F n t}{\mathrm{d} a} = F n t, \qquad \frac{r \, \mathrm{d} R}{\mathrm{d} r} = a \left(\frac{\mathrm{d} R}{\mathrm{d} a}\right),$$

$$\{a^2 - 2 a a_i \cos\theta + a_i^2\}^{-\frac{1}{2}} = \frac{1}{a_i}\{b_{1,0} + b_{1,1}\cos\theta + b_{1,2}\cos 2\theta + \&c.\}$$

$$\frac{-(a - a_i\cos\theta)}{\{a^2 - 2 a a_i \cos\theta + a_i^2\}^{\frac{3}{2}}} = \frac{1}{a_i}\left\{\frac{\mathrm{d} . b_{1,0}}{\mathrm{d} a} + \frac{\mathrm{d} b_{1,1}}{\mathrm{d} a}\cos\theta + \frac{\mathrm{d} . b_{1,2}}{\mathrm{d} a}\cos 2\theta + \&c.\right\}$$

$$\frac{-\{a - a_i\cos\theta\}}{a_i^3}\{b_{3,0} + b_{3,1}\cos\theta + b_{3,2}\cos 2\theta + \&c.\} = \frac{1}{a_i}\left\{\frac{\mathrm{d} . b_{1,0}}{\mathrm{d} a}\right.$$

$$\left. + \frac{\mathrm{d} . b_{1,1}}{\mathrm{d} a}\cos\theta + \frac{\mathrm{d} . b_{1,2}}{\mathrm{d} a}\cos 2\theta + \&c.\right\}$$

whence

$$\frac{a \, \mathrm{d} . b_{1,0}}{\mathrm{d} a} = -\frac{a}{a_i}\left\{\frac{a}{a_i} b_{3,0} - \frac{1}{2} b_{3,1}\right\}$$

$$\frac{a \, \mathrm{d} . b_{1,1}}{\mathrm{d} a} = -\frac{a}{a_i}\left\{\frac{a}{a_i} b_{3,1} - b_{3,0} - \frac{1}{2} b_{3,2}\right\}$$

$$\frac{a \, \mathrm{d} . b_{1,2}}{\mathrm{d} a} = -\frac{a}{a_i}\left\{\frac{a}{a_i} b_{3,2} - \frac{1}{2} b_{3,1} - \frac{1}{2} b_{3,3}\right\}$$

$$\frac{a \, \mathrm{d} . b_{1,3}}{\mathrm{d} a} = -\frac{a}{a_i}\left\{\frac{a}{a_i} b_{3,3} - \frac{1}{2} b_{3,2} - \frac{1}{2} b_{3,4}\right\}$$

similarly

$$\frac{a \, \mathrm{d} . b_{3,0}}{\mathrm{d} a} = -3\frac{a}{a_i}\left\{\frac{a}{a} b_{5,0} - \frac{1}{2} b_{5,1}\right\}$$

$$\frac{a \, \mathrm{d} . b_{3,1}}{\mathrm{d} a} = -3\frac{a}{a_i}\left\{\frac{a}{a_i} b_{5,1} - b_{5,0} - \frac{1}{2} b_{5,1}\right\}$$

$$\frac{a \, \mathrm{d} . b_{5,0}}{\mathrm{d} a} = -5\frac{a}{a_i}\left\{\frac{a}{a_i} b_{7,0} - \frac{1}{2} b_{7,1}\right\}$$

$$\frac{a \, \mathrm{d} . b_{5,1}}{\mathrm{d} a} = -5\frac{a}{a_i}\left\{\frac{a}{a_i} b_{7,1} - b_{7,0} - \frac{1}{2} b_{7,2}\right\}$$

$$\frac{a^2\,\mathrm{d}^2.\,b_{1,0}}{\mathrm{d}a^2}=\frac{a}{a_{\prime}}\left\{\frac{2\,a}{a_{\prime}}b_{3,0}-\frac{1}{2}\,b_{3,1}\right\}$$

$$\frac{a^2\,\mathrm{d}^2.\,b_{1,1}}{\mathrm{d}a^2}=\frac{a}{a_{\prime}}\left\{\frac{2\,a}{a_{\prime}}b_{3,1}-b_{3,2}\right\}$$

$$\frac{a^2\,\mathrm{d}^2.\,b_{1,2}}{\mathrm{d}a^2}=\frac{a}{a_{\prime}}\left\{\frac{2\,a}{a_{\prime}}b_{3,2}+\frac{1}{2}b_{3,1}-\frac{3}{2}\,b_{3,3}\right\}$$

$$\frac{a^2\,\mathrm{d}^2.\,b_{1,3}}{\mathrm{d}a^2}=\frac{a}{a_{\prime}}\left\{\frac{2\,a}{a_{\prime}}b_{3,3}+b_{3,2}-2b_{3,4}\right\}$$

$$\frac{a^2\,\mathrm{d}^2.\,b_{1,4}}{\mathrm{d}a^2}=\frac{a}{a_{\prime}}\left\{\frac{2\,a}{a_{\prime}}b_{3,4}+\frac{3}{2}\,b_{3,3}-\frac{5}{2}\,b_{3,5}\right\}$$

The value of R given p. 349 of the former part of this paper is susceptible of much simplification. The first term of R for instance

$$=m_{\prime}\left\{-\frac{b_{1,0}}{a_{\prime}}+\frac{3\,(a^2e^2+a_{\prime}^2e_{\prime}^2)}{2\,.\,2\,a_{\prime}^3}\,b_{3,0}+\frac{a\,a_{\prime}}{2\,a_{\prime}^3}\left(\sin^2\frac{\iota_{\prime}}{2}+\frac{(e^2+e_{\prime}^2)}{2}\right)b_{3,1}\right.$$
$$-\frac{3}{2\,.\,4\,a_{\prime}^5}\left(2\,a^4e^2+5\,a^2a_{\prime}^2\,(e^2+e_{\prime}^2)+2\,a_{\prime}^4e_{\prime}^2\right)b_{5,0}+\frac{3\,.\,2}{2\,.\,4\,a_{\prime}^5}\,(a^2e^2+a_{\prime}^2e_{\prime}^2)\,a\,a_{\prime}\,b_{5,1}$$
$$\left.+\frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2a_{\prime}^2}{a_{\prime}^5}\,(e^2+e_{\prime}^2)\,b_{5,2}\right\}$$

$$=m_{\prime}\left\{-\frac{b_{1,0}}{a_{\prime}}+\frac{3\,(a^2e^2+a_{\prime}^2e_{\prime}^2)}{2\,.\,2\,a_{\prime}^3}\,b_{3,0}+\frac{a\,a_{\prime}}{2a_{\prime}^3}\left(\sin^2\frac{\iota_{\prime}}{2}+\frac{e^2+e_{\prime}^2}{2}\right)b_{3,1}\right.$$
$$-\frac{3\,.\,2}{2\,.\,4}\,\frac{(a^2+a_{\prime}^2)}{a_{\prime}^2}\,\frac{(a^2e^2+a_{\prime}^2e_{\prime}^2)}{a_{\prime}^3}\,b_{5,0}-\frac{3\,.\,3}{2\,.\,4}\,\frac{a^2a_{\prime}^2}{a_{\prime}^5}\,(e^2+e_{\prime}^2)\,b_{5,0}$$
$$\left.+\frac{3\,.\,2}{2\,.\,4}\,\frac{(a^2e^2+a_{\prime}^2e_{\prime}^2)}{a_{\prime}^3}\,\frac{a}{a_{\prime}}\,b_{5,1}+\frac{3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2a_{\prime}^2}{a_{\prime}^5}\,(e^2+e_{\prime}^2)\,b_{5,2}\right\}$$

and since

$$b_{3,0}=\frac{(a^2+a_{\prime}^2)}{a_{\prime}^2}\,b_{5,0}-\frac{a}{a_{\prime}}\,b_{5,1}\quad\text{See p. 26}$$

$$b_{3,1}=\frac{3\,a}{a_{\prime}}\left\{b_{5,0}-\tfrac{1}{2}\,b_{5,2}\right\}$$

this term reduces itself to

$$m_{\prime}\left\{-\frac{b_{1,0}}{a_{\prime}}+\frac{a\,a_{\prime}}{2\,a_{\prime}^3}\left(\sin^2\frac{\iota_{\prime}}{2}+\frac{e^2+e_{\prime}^2}{2}\right)b_{3,1}-\frac{3}{2\,.\,4}\,\frac{a\,a_{\prime}}{a_{\prime}^3}\,(e^2+e_{\prime}^2)\,b_{3,1}\right\}$$
$$=m_{\prime}\left\{-\frac{b_{1,0}}{a_{\prime}}+\frac{a}{2\,a_{\prime}^2}\left(\sin^2\frac{\iota_{\prime}}{2}-\frac{e^2+e_{\prime}^2}{4}\right)b_{3,1}\right\}$$

The succeeding terms admit of similar simplifications, so that

$$R = m_{,}\left\{-\frac{b_{1,0}}{a_{,}} + \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} + \frac{e^2+e_{,}^2}{2}\right)b_{3,1} - \frac{3}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,1}\right\}$$

$$+ m_{,}\left\{\frac{a}{a_{,}^2}\left(\cos^2\frac{\iota_{,}}{2} - \frac{e^2+e_{,}^2}{2}\right) - \frac{b_{1,1}}{a_{,}} + \frac{a}{a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} + \frac{e^2+e_{,}^2}{2}\right)\left(b_{3,0} + \tfrac{1}{2}b_{3,2}\right)\right.$$

$$\left. - \frac{3\,.\,2}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)b_{3,2}\right\}\cos(nt - n_{,}t + \varepsilon - \varepsilon_{,}) \qquad [1]$$

$$+ m_{,}\left\{-\frac{b_{1,2}}{a_{,}} + \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} + \frac{e^2+e_{,}^2}{2}\right)(b_{3,1}+b_{3,3}) + \frac{3}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,1}\right.$$

$$\left. - \frac{3\,.\,3}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,3}\right\}\cos(2nt - 2n_{,}t + 2\varepsilon - 2\varepsilon_{,}) \qquad [2]$$

$$+ m_{,}\left\{-\frac{b_{1,3}}{a_{,}} + \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} + \frac{e^2+e_{,}^2}{2}\right)(b_{3,2}+b_{3,4}) + \frac{3\,.\,2}{3\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,2}\right.$$

$$\left. - \frac{3\,.\,4}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,4}\right\}\cos(3nt - 3n_{,}t - 3\varepsilon - 3\varepsilon_{,}) \qquad [3]$$

$$+ m_{,}\left\{-\frac{b_{1,4}}{a_{,}} + \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} + \frac{e^2+e_{,}^2}{2}\right)(b_{3,3}+b_{3,5}) + \frac{3\,.\,3}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)b_{3,3}\right.$$

$$\left. - \frac{3\,.\,5}{2\,.\,4}\frac{a}{a_{,}^2}(e^2+e_{,}^2)\,b_{3,5}\right\}\cos(4nt - 4n_{,}t + 4\varepsilon - 4\varepsilon_{,}) \qquad [4]$$

The coefficient of $\cos(\varpi - \varpi_{,})$, Argument 41,

$$= m_{,}\left\{-\frac{9}{4}\frac{a}{a_{,}^3}b_{3,0} - \frac{a}{8a_{,}^2}b_{3,2} + \frac{3\,.\,6}{2\,.\,4}\frac{(a^2+a_{,}^2)}{a_{,}^5}a\,a_{,}\,b_{5,0} - \frac{3\,.\,7}{2\,.\,4\,.\,2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}\right.$$

$$\left. - \frac{3}{2\,.\,4}\frac{(a^2+a_{,}^2)}{a_{,}^5}a\,a_{,}\,b_{5,2} - \frac{3}{2\,.\,4\,.\,2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\right\}e\,e_{,}$$

$$= m_{,}\left\{-\frac{9}{4}\frac{a}{a_{,}^2}b_{3,0} - \frac{a}{8a_{,}^2}b_{3,2} + \frac{3\,.\,6}{2\,.\,4}\frac{a}{a_{,}^2}\left\{\frac{(a^2+a_{,}^2)}{a_{,}^2}b_{5,0} - \frac{a}{a_{,}}b_{5,1}\right\}\right.$$

$$\left. - \frac{3}{8}\frac{a}{a_{,}^2}\left\{\frac{(a^2+a_{,}^2)}{a_{,}^2}b_{5,2} - \frac{a}{a_{,}}b_{5,1} - \frac{a}{a_{,}}b_{5,3}\right\} + \frac{9}{16}\frac{a}{a_{,}^3}\left\{b_{5,1} - b_{5,3}\right\}\right\}$$

$$= m_{,}\left\{-\frac{9}{4}\frac{a}{a_{,}^2}b_{3,0} - \frac{a}{8a_{,}^2}b_{3,2} + \frac{9}{4}\frac{a}{a_{,}^2}b_{3,0} - \frac{3}{8}\frac{a}{a_{,}^2}b_{3,2} + \frac{2\,.\,3}{8}\frac{a}{a_{,}^2}b_{3,2}\right\}e\,e_{,}$$

$$= m_{,}\frac{a}{4a_{,}^2}b_{3,2}$$

So that the part of R which is independent of $n\,t$, $n_{,}\,t$

$$= m_{,}\left\{-\frac{b_{1,0}}{a_{,}^2} + \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} - \frac{e^2+e_{,}^2}{4}\right)b_{3,1} + \frac{a}{4a_{,}^2}b_{3,2}\,e\,e_{,}\cos(\varpi - \varpi_{,})\right\}$$

$$= m_{,}\left\{-\frac{b_{1,0}}{a_{,}} - \frac{a}{2a_{,}^2}\left(\sin^2\frac{\iota_{,}}{2} - \frac{e^2+e_{,}^2}{4}\right)b_{3,1} - \left\{\frac{3a}{2a_{,}^2}b_{3,0} - \frac{a^2+a_{,}^2}{2a_{,}^3}b_{3,1}\right\}e\,e_{,}\cos(\varpi - \varpi_{,})\right\}$$

In the general case, when ι_1, ι_2 are the inclinations of the orbits of the planets P and $P_{\prime}$ to any plane, the direction of which is arbitrary,

$$\cos \iota_{\prime} = \cos \iota_1 \cos \iota_2 + \sin \iota_1 \sin \iota_2 \cos (\nu_1 - \nu_2)$$

$$\sin^2 \frac{\iota_{\prime}}{2} = \frac{1 - \cos \iota_1 \cos \iota_2 - \sin \iota_1 \sin \iota_2 \cos (\nu_1 - \nu_2)}{2}$$

The part of R which is independent of $n\,t$, $n_{\prime}t$

$$= m_{\prime}\left\{ -\frac{b_{1,0}}{a_{\prime}} + \frac{a}{4\,a_{\prime}^2}\left\{1 - \cos \iota_1 \cos \iota_2 - \sin \iota_1 \sin \iota_2 \cos (\nu_1 - \nu_2) - \frac{e^2 + e_{\prime}^2}{2}\right\} b_{3,1} \right.$$
$$\left. - \left\{\frac{3\,a}{2\,a_{\prime}^2} b_{3,0} - \frac{(a^2 + a_{\prime}^2)}{2\,a_{\prime}^3} b_{3,1}\right\} e\,e_{\prime} \cos (\varpi - \varpi_{\prime}) \right\}$$

$$\cos \iota = \frac{1}{\sqrt{1 + \tan^2 \iota}} = 1 - \tfrac{1}{2} \tan^2 \iota, \qquad \sin \iota = \frac{\tan \iota}{\sqrt{1 + \tan^2 \iota}} = \tan \iota \text{ nearly.}$$

$$= m_{\prime}\left\{ -\frac{b_{1,0}}{a_{\prime}} + \frac{a}{8\,a_{\prime}^2}\left\{(\tan^2 \iota_1 + \tan^2 \iota_2 - 2 \tan \iota_1 \tan \iota_2 \cos (\nu_1 - \nu_2) - e^2 - e_{\prime}^2\right\} b_{3,1} \right.$$
$$\left. - \left\{\frac{3\,a}{2\,a_{\prime}^2} b_{3,0} - \frac{(a^2 + a_{\prime}^2)}{2\,a_{\prime}^3} b_{3,1}\right\} e\,e_{\prime} \cos (\varpi - \varpi_{\prime}) \right\}$$

$$= m_{\prime}\left\{ -\frac{b_{1,0}}{a_{\prime}} + \frac{a}{8\,a_{\prime}^2}\left\{(\tan \iota_1 \cos \nu_1 - \tan \iota_2 \cos \nu_2)^2 + (\tan \iota_1 \sin \nu_1 - \tan \iota_2 \sin \iota_2)^2 - e^2 - e_{\prime}^2\right\} b_{3,1} \right.$$
$$\left. - \left\{\frac{3\,a}{2\,a_{\prime}^2} b_{3,0} - \frac{(a^2 + a_{\prime}^2)}{2\,a_{\prime}^3} b_{3,1}\right\} e\,e_{\prime} \cos (\varpi - \varpi_{\prime}) \right\}$$

and if $\tan \iota \sin \nu = p$, $\tan \iota \cos \nu = q$, this quantity

$$= m_{\prime}\left\{ -\frac{b_{1,0}}{a_{\prime}} + \frac{a}{8\,a_{\prime}^2}\left\{(p_1 - p_2)^2 + (q_1 - q_2)^2 - e^2 - e_{\prime}^2\right\} b_{3,1} \right.$$
$$\left. - \left\{\frac{3\,a}{2\,a_{\prime}^2} b_{3,0} - \frac{a^2 + a_{\prime}^2}{2\,a_{\prime}^3} b_{3,1}\right\} e\,e_{\prime} \cos (\varpi - \varpi_{\prime}) \right\}$$

which evidently agrees with the result given by M. de PONTÉCOULANT, Théor. Anal. vol. i. p. 363. All the other coefficients of terms multiplied by the squares and products of the eccentricities are susceptible of reductions similar to those in the two preceding pages, and finally;

$$R = m_{\prime}\left\{ -\frac{b_{1,0}}{a_{\prime}} + \frac{a}{2\,a_{\prime}^2}\left(\sin^2 \frac{\iota_{\prime}}{2} - \frac{e^2 + e_{\prime}^2}{4}\right) b_{3,1} \right\} \qquad [0]$$

$$+ m_{\prime}\left\{ -\frac{a}{a_{\prime}^2}\left(\cos^2 \frac{\iota_{\prime}}{2} - \frac{e^2 + e_{\prime}^2}{2}\right) - \frac{b_{1,1}}{a_{\prime}} + \frac{a}{a_{\prime}^2} \sin^2 \frac{\iota_{\prime}}{2} (b_{3,0} + \tfrac{1}{2} b_{3,2}) \right.$$
$$\left. + \frac{a}{a_{\prime}^2} \frac{(e^2 + e_{\prime}^2)}{8} (4\,b_{3,0} - 4\,b_{3,2}) \right\} \cos (n\,t - n_{\prime}\,t + \varepsilon - \varepsilon_{\prime}) \qquad [1]$$

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$$+ m_{,}\left\{ -\frac{b_{1,2}}{a_{,}} + \frac{a}{2\,a_{,}^{2}} \sin^{2}\frac{\iota_{,}}{2}\,(b_{3,1} + b_{3,3}) + \frac{a}{a_{,}^{2}}\,\frac{(e^{2} + e_{,}^{2})}{8}(5\,b_{3,1} - 7\,b_{3,3}) \right\} \cos\,(2\,n\,t - 2\,n_{,}\,t + 2\,\varepsilon - 2\,\varepsilon_{,})$$ [2]

$$+ m_{,}\left\{ -\frac{b_{1,3}}{a_{,}} + \frac{a}{2\,a_{,}^{2}} \sin^{2}\frac{\iota_{,}}{2}\,(b_{3,2} + b_{3,4}) + \frac{a}{a_{,}^{2}}\,\frac{(e^{2} + e_{,}^{2})}{8}\,(8\,b_{3,2} - 10\,b_{3,4}) \right\} \cos\,(3\,n\,t - 3\,n_{,}\,t + 3\,\varepsilon - 3\,\varepsilon_{,})$$ [3]

$$+ m_{,}\left\{ -\frac{b_{1,4}}{a_{,}} + \frac{a}{2\,a_{,}^{2}} \sin^{2}\frac{\iota_{,}}{2}\,(b_{3,3} + b_{3,5}) + \frac{a}{a_{,}^{2}}\,\frac{(e^{2} + e_{,}^{2})}{8}\,(11\,b_{3,3} - 13\,b_{3,5}) \right\} \cos\,(4\,n\,t - 4\,n_{,}\,t + 4\,\varepsilon - 4\,\varepsilon_{,})$$ [4]

$$+ m_{,}\left\{ -\frac{b_{1,5}}{a_{,}} + \frac{a}{2\,a_{,}^{2}} \sin^{2}\frac{\iota_{,}}{2}\,(b_{3,4} + b_{3,6}) + \frac{a}{a_{,}^{2}}\,\frac{(e^{2} + e_{,}^{2})}{8}\,(14\,b_{3,4} - 16\,b_{3,6}) \right\} \cos\,(5\,n\,t_{,} - 5\,n_{,}\,t + 5\,\varepsilon - 5\,\varepsilon_{,})$$ [5]

$$+ m_{,}\left\{ -\frac{3\,a}{2\,a_{,}^{3}} + \frac{3\,a}{2\,a_{,}^{2}}\,b_{3,0} - \frac{a^{2}}{2\,a_{,}^{3}}\,b_{3,1} - \frac{a}{4\,a_{,}^{2}}\,b_{3,2} \right\} e\cos\,(n_{,}\,t + \varepsilon - \varpi)$$ [6] [16]*

$$+ m_{,}\left\{ -\frac{a^{2}}{a_{,}^{3}}\,b_{3,0} + \frac{a}{2\,a_{,}^{2}}\,b_{3,1} \right\} e\cos\,(n\,t + \varepsilon - \varpi)$$ [7] [15]

$$+ m_{,}\left\{ \frac{a}{2\,a_{,}^{2}} - \frac{a}{2\,a_{,}^{2}}\,b_{3,0} - \frac{a^{2}}{2\,a_{,}^{3}}\,b_{3,1} + \frac{3}{4}\,\frac{a}{a_{,}^{2}}\,b_{3,2} \right\} e\cos\,(2\,n\,t - n_{,}\,t + 2\,\varepsilon - \varepsilon_{,} - \varpi)$$ [8] [20]

$$+ m_{,}\left\{ -\frac{a}{4\,a_{,}^{2}}\,b_{3,1} - \frac{a^{2}}{2\,a_{,}^{2}}\,b_{3,2} + \frac{3}{4}\,\frac{a}{a_{,}^{2}}\,b_{3,3} \right\} e\cos\,(3\,n\,t - 2\,n_{,}\,t + 3\,\varepsilon - 2\,\varepsilon_{,} - \varpi)$$ [9] [21]

$$+ m_{,}\left\{ -\frac{a}{4\,a_{,}^{2}}\,b_{3,2} - \frac{a^{2}}{2\,a_{,}^{3}}\,b_{3,3} + \frac{3}{4}\,\frac{a}{a_{,}^{2}}\,b_{3,4} \right\} e\cos\,(4\,n\,t - 3\,n_{,}\,t + 4\,\varepsilon - 3\,\varepsilon_{,} - \varpi)$$ [10] [22]

$$+ m_{,}\left\{ -\frac{a}{4\,a_{,}^{2}}\,b_{3,3} - \frac{a}{2\,a_{,}^{3}}\,b_{3,4} + \frac{3}{4}\,\frac{a}{a_{,}^{2}}\,b_{3,5} \right\} e\cos\,(5\,n\,t - 4\,n_{,}\,t + 5\,\varepsilon - 4\,\varepsilon_{,} - \varpi)$$ [11] [23]

$$+ m_{,}\left\{ \frac{3\,a}{4\,a_{,}^{2}}\,b_{3,1} - \frac{a^{2}}{2\,a_{,}^{3}}\,b_{3,2} - \frac{a}{4\,a_{,}^{2}}\,b_{3,3} \right\} e\cos\,(n\,t - 2\,n_{,}\,t + \varepsilon - 2\,\varepsilon_{,} + \varpi)$$ [12] [17]

$$+ m_{,}\left\{ \frac{3\,a}{4\,a_{,}^{2}}\,b_{3,2} - \frac{a^{2}}{2\,a_{,}^{2}}\,b_{3,3} - \frac{a}{4\,a_{,}^{2}}\,b_{3,4} \right\} e\cos\,(2\,n\,t - 3\,n_{,}\,t + 2\,\varepsilon - 3\,\varepsilon_{,} + \varpi)$$ [13] [18]

$$+ m_{,}\left\{ \frac{3\,a}{4\,a_{,}^{2}}\,b_{3,3} - \frac{a^{2}}{2\,a_{,}^{3}}\,b_{3,4} - \frac{a}{4\,a_{,}^{2}}\,b_{3,5} \right\} e_{,}\cos\,(3\,n\,t - 4\,n_{,}t + 3\,\varepsilon - 4\,\varepsilon_{,} + \varpi)$$ [14] [19]

$$+ m_{,}\left\{ -\frac{a_{,}^{2}}{a_{,}^{3}}\,b_{3,0} + \frac{a}{2\,a_{,}^{2}}\,b_{3,1} \right\} e_{,}\cos\,(n_{,}t + \varepsilon_{,} - \varpi_{,})$$ [15] [7]

* These numbers indicate the arguments which are symmetrical with regard to $n\,t$ and $n_{,}\,t$.

Development of R.

$$+ m_{\prime} \left\{ \frac{3\,a}{2\,a_{\prime}^2} b_{3,0} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,1} - \frac{a}{4\,a_{\prime}^2} b_{3,2} \right\} e_{\prime} \cos (n\,t + \varepsilon - \varpi_{\prime}) \qquad [16] \quad [6]$$

$$+ m_{\prime} \left\{ \frac{3\,a}{4\,a_{\prime}^2} b_{3,1} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,2} - \frac{a}{4\,a_{\prime}^2} b_{3,3} \right\} e_{\prime} \cos (2\,n\,t - n_{\prime}t + 2\,\varepsilon - \varepsilon_{\prime} - \varpi_{\prime}) \qquad [17] \quad [12]$$

$$+ m_{\prime} \left\{ \frac{3\,a}{4\,a_{\prime}^2} b_{3,2} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,3} - \frac{a}{4\,a_{\prime}^2} b_{3,4} \right\} e_{\prime} \cos (3\,n\,t - 2\,n_{\prime}t + 3\,\varepsilon - 2\,\varepsilon_{\prime} - \varpi_{\prime}) \qquad [18] \quad [13]$$

$$+ m_{\prime} \left\{ \frac{3\,a}{4\,a_{\prime}^2} b_{3,3} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,4} - \frac{a}{4\,a_{\prime}^2} b_{3,5} \right\} e_{\prime} \cos (4\,n\,t - 3\,n_{\prime}t + 4\,\varepsilon - 3\,\varepsilon_{\prime} - \varpi_{\prime}) \qquad [19] \quad [14]$$

$$+ m_{\prime} \left\{ \frac{a}{2\,a_{\prime}^2} - \frac{a}{2\,a_{\prime}^2} b_{3,0} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,1} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,2} \right\} e_{\prime} \cos (n\,t - 2\,n_{\prime}t + \varepsilon - 2\,\varepsilon_{\prime} + \varpi_{\prime}) \qquad [20] \quad [8]$$

$$+ m_{\prime} \left\{ - \frac{a}{4\,a_{\prime}^2} b_{3,1} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,2} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,3} \right\} e_{\prime} \cos (2\,n\,t - 3\,n_{\prime}t + 2\,\varepsilon - 3\,\varepsilon_{\prime} + \varpi_{\prime}) \qquad [21] \quad [9]$$

$$+ m_{\prime} \left\{ - \frac{a}{4\,a_{\prime}^2} b_{3,2} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,3} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,4} \right\} e_{\prime} \cos (3\,n\,t - 4\,n_{\prime}t + 3\,\varepsilon - 4\,\varepsilon_{\prime} + \varpi_{\prime}) \qquad [22] \quad [10]$$

$$+ m_{\prime} \left\{ - \frac{a}{4\,a_{\prime}^2} b_{3,3} - \frac{a_{\prime}^2}{2\,a_{\prime}^3} b_{3,4} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,5} \right\} e_{\prime} \cos (4\,n\,t - 5\,n_{\prime}t + 4\,\varepsilon - 5\,\varepsilon_{\prime} + \varpi_{\prime}) \qquad [23] \quad [11]$$

$$+ m_{\prime} \left\{ - \frac{5}{8} \frac{a}{a_{\prime}^2} b_{3,1} + \frac{a^2}{2\,a_{\prime}^3} b_{3,2} \right\} e^2 \cos (2 n_{\prime}t + 2\,\varepsilon_{\prime} - 2\,\varpi) \qquad [24] \quad [59]$$

$$+ m_{\prime} \left\{ \frac{a}{8\,a_{\prime}^2} - \frac{a}{8\,a_{\prime}^2} b_{3,0} - \frac{a}{16\,a_{\prime}^2} b_{3,2} \right\} e^2 \cos (n\,t + n_{\prime}t + \varepsilon + \varepsilon_{\prime} - 2\,\varpi) \qquad [25] \quad [58]$$

$$+ m_{\prime} \left\{ - \frac{a^2}{a_{\prime}^3} b_{3,0} + \frac{3}{8} \frac{a}{a_{\prime}^2} b_{3,1} \right\} e^2 \cos (2\,n\,t + 2\,\varepsilon - 2\,\varpi) \qquad [26] \quad [57]$$

$$+ m_{\prime} \left\{ \frac{3}{8} \frac{a}{a_{\prime}^2} - \frac{3}{8} \frac{a}{a_{\prime}^2} b_{3,0} - \frac{a^2}{a_{\prime}^3} b_{3,1} + \frac{17}{16} \frac{a}{a_{\prime}^2} b_{3,2} \right\} e^2 \cos (3\,n\,t - n_{\prime}t + 3\,\varepsilon - \varepsilon_{\prime} - 2\,\varpi) \qquad [27] \quad [63]$$

$$+ m_{\prime} \left\{ - \frac{a}{4\,a_{\prime}^2} b_{3,1} - \frac{3}{2} \frac{a^2}{a_{\prime}^3} b_{3,2} + \frac{13}{8} \frac{a}{a_{\prime}^2} b_{3,3} \right\} e^2 \cos (4\,n\,t - 2\,n_{\prime}t + 4\,\varepsilon - 2\,\varepsilon_{\prime} - 2\,\varpi) \qquad [28] \quad [64]$$

$$+ m_{\prime} \left\{ - \frac{5}{16} \frac{a}{a_{\prime}^2} b_{3,2} - 2\, \frac{a^2}{a_{\prime}^3} b_{3,3} + \frac{35}{16} \frac{a}{a_{\prime}^2} b_{3,4} \right\} e^2 \cos (5\,n\,t - 3\,n_{\prime}t + 5\,\varepsilon - 3\,\varepsilon_{\prime} - 2\,\varpi) \qquad [29] \quad [65]$$

$$+ m_{\prime} \left\{ - \frac{3}{8} \frac{a}{a_{\prime}^2} b_{3,3} - \frac{5}{2} \frac{a^2}{a_{\prime}^3} b_{3,4} + \frac{11}{4} \frac{a}{a_{\prime}^2} b_{3,5} \right\} e^2 \cos (6\,n\,t - 4\,n_{\prime}t + 6\,\varepsilon - 4\,\varepsilon_{\prime} - 2\,\varpi) \qquad [30] \quad [66]$$

$$+ m_{\prime} \left\{ - \frac{7}{16} \frac{a}{a_{\prime}^2} b_{3,4} - 3\, \frac{a^2}{a_{\prime}^3} b_{3,5} + \frac{53}{16} \frac{a}{a_{\prime}^2} b_{3,6} \right\} e^2 \cos (7\,n\,t - 5\,n_{\prime}t + 7\,\varepsilon - 5\,\varepsilon_{\prime} - 2\,\varpi) \qquad [31] \quad [67]$$

$$+ m_{\prime} \left\{ - \frac{19}{16} \frac{a}{a_{\prime}^2} b_{3,2} + \frac{a^2}{a_{\prime}^3} b_{3,3} + \frac{a}{16\,a_{\prime}^2} b_{3,4} \right\} e^2 \cos (n\,t - 3\,n_{\prime}t + \varepsilon - 3\,\varepsilon_{\prime} + 2\,\varpi) \qquad [32] \quad [60]$$

$$+ m_{\prime} \left\{ - \frac{7}{4} \frac{a}{a_{\prime}^2} b_{3,3} + \frac{3}{2} \frac{a^2}{a_{\prime}^3} b_{3,4} + \frac{1}{8} \frac{a}{a_{\prime}^2} b_{3,5} \right\} e^2 \cos (2\,n\,t - 4\,n_{\prime}t + 2\,\varepsilon - 4\,\varepsilon_{\prime} + 2\,\varpi) \qquad [33] \quad [61]$$

Development of R.

$$+ m_{\prime} \left\{ - \frac{37}{16} \frac{a}{a_{\prime}^{2}} b_{3,4} + 2 \frac{a^{2}}{a_{\prime}^{3}} b_{3,5} + \frac{3}{16} \frac{a}{a_{\prime}^{2}} b_{3,6} \right\} e^{2} \cos (3nt - 5n_{\prime}t + 3\varepsilon - 5\varepsilon_{\prime} + 2\varpi)$$ [34] [62]

$$+ m_{\prime} \frac{a}{4a_{\prime}^{2}} b_{3,1} e e_{\prime} \cos (nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime} - \varpi + \varpi_{\prime})$$ [35]

$$+ m_{\prime} \left\{ \frac{a}{a_{\prime}^{2}} - \frac{a}{a_{\prime}^{2}} b_{3,0} + \frac{3}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} \right\} e e_{\prime} \cos (2nt - 2n_{\prime}t + 2\varepsilon - 2\varepsilon_{\prime} - \varpi + \varpi_{\prime})$$ [36]

$$- m_{\prime} \left\{ \frac{7}{8} \frac{a}{a_{\prime}^{2}} b_{3,1} - \frac{9}{8} \frac{a}{a_{\prime}^{2}} b_{3,3} \right\} e e_{\prime} \cos (3nt - 3n_{\prime}t + 3\varepsilon - 3\varepsilon_{\prime} - \varpi + \varpi_{\prime})$$ [37]

$$- m_{\prime} \left\{ \frac{5}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} - \frac{3}{2} \frac{a}{a_{\prime}^{2}} b_{3,4} \right\} e e_{\prime} \cos (4nt - 4n_{\prime}t + 4\varepsilon - 4\varepsilon_{\prime} - \varpi + \varpi_{\prime})$$ [38]

$$- m_{\prime} \left\{ \frac{13}{8} \frac{a}{a_{\prime}^{2}} b_{3,3} - \frac{15}{8} \frac{a}{a_{\prime}^{2}} b_{3,5} \right\} e e_{\prime} \cos (5nt - 5n_{\prime}t + 5\varepsilon - 5\varepsilon_{\prime} - \varpi + \varpi_{\prime})$$ [39]

$$- m_{\prime} \left\{ 2 \frac{a}{a_{\prime}^{2}} b_{3,4} - \frac{9}{4} \frac{a}{a_{\prime}^{2}} b_{3,6} \right\} e e_{\prime} \cos (6nt - 6n_{\prime}t + 6\varepsilon - 6\varepsilon_{\prime} - \varpi - \varpi_{\prime})$$ [40]

$$+ m_{\prime} \frac{a}{4a_{\prime}^{2}} b_{3,2} e e_{\prime} \cos (\varpi - \varpi_{\prime})$$ [41]

$$- m_{\prime} \left\{ \frac{3}{8} \frac{a}{a_{\prime}^{2}} b_{3,1} - \frac{5}{8} \frac{a}{a_{\prime}^{2}} b_{3,3} \right\} e e_{\prime} \cos (nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime} + \varpi - \varpi_{\prime})$$ [42]

$$- m_{\prime} \left\{ \frac{3}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} - \frac{a}{a_{\prime}^{2}} b_{3,4} \right\} e e_{\prime} \cos (2nt - 2n_{\prime}t + 2\varepsilon - 2\varepsilon_{\prime} + \varpi - \varpi_{\prime})$$ [43]

$$- m_{\prime} \left\{ \frac{9}{8} \frac{a}{a_{\prime}^{2}} b_{3,3} - \frac{11}{8} \frac{a}{a_{\prime}^{2}} b_{3,5} \right\} e e_{\prime} \cos (3nt - 3n_{\prime}t + 3\varepsilon - 3\varepsilon_{\prime} + \varpi - \varpi_{\prime})$$ [44]

$$- m_{\prime} \left\{ \frac{3}{2} \frac{a}{a_{\prime}^{2}} b_{3,4} - \frac{7}{4} \frac{a}{a_{\prime}^{2}} b_{3,6} \right\} e e_{\prime} \cos (4nt - 4n_{\prime}t + 4\varepsilon - 4\varepsilon_{\prime} + \varpi - \varpi_{\prime})$$ [45]

$$+ m' \left\{ - \frac{3a}{a_{\prime}^{2}} + \frac{1}{a_{\prime}} b_{3,1} - \frac{3}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} \right\} e e_{\prime} \cos (2n_{\prime}t + 2\varepsilon_{\prime} - \varpi - \varpi_{\prime})$$ [46] [48]

$$+ m_{\prime} \frac{a}{4a_{\prime}^{2}} b_{3,1} e e_{\prime} \cos (nt + n_{\prime}t + \varepsilon + \varepsilon_{\prime} - \varpi - \varpi_{\prime})$$ [47]

$$+ m_{\prime} \left\{ \frac{a^{2}}{a_{\prime}^{3}} b_{3,1} - \frac{3}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} \right\} e e_{\prime} \cos (2nt + 2\varepsilon - \varpi - \varpi_{\prime})$$ [48] [46]

$$+ m_{\prime} \left\{ \frac{21}{8} \frac{a}{a_{\prime}^{2}} b_{3,1} - \frac{2}{a_{\prime}} b_{3,2} - \frac{3}{8} \frac{a}{a_{\prime}^{2}} b_{3,3} \right\} e e_{\prime} \cos (3nt - n_{\prime}t + 3\varepsilon - \varepsilon_{\prime} - \varpi - \varpi)$$ [49] [53]

$$+ m_{\prime} \left\{ \frac{15}{4} \frac{a}{a_{\prime}^{2}} b_{3,2} - \frac{3}{a_{\prime}} b_{3,3} - \frac{1}{2} \frac{a}{a_{\prime}^{2}} b_{3,4} \right\} e e_{\prime} \cos (4nt - 2n_{\prime}t + 4\varepsilon - 2\varepsilon_{\prime} - \varpi - \varpi)$$ [50] [54]

Development of R.

$$+ m_{,} \left\{ \frac{39}{8} \frac{a}{a_{,}^2} b_{3,3} - \frac{4}{a_{,}} b_{3,4} - \frac{5}{8} \frac{a}{a_{,}^2} b_{3,5} \right\} e e_{,} \cos (5 n t - 3 n_{,} t + 5 \varepsilon - 3 \varepsilon_{,} - \varpi - \varpi_{,}) \quad [51] \quad [55]$$

$$+ m_{,} \left\{ 6 \frac{a}{a_{,}^2} b_{3,4} - \frac{5}{a_{,}} b_{3,5} - \frac{3}{4} \frac{a}{a_{,}^2} b_{3,6} \right\} e e_{,} \cos (6 n t - 4 n_{,} t + 6 \varepsilon - 4 \varepsilon_{,} - \varpi - \varpi_{,}) \quad [52] \quad [56]$$

$$+ m_{,} \left\{ \frac{21}{8} \frac{a}{a_{,}^2} b_{3,1} - \frac{2 a^2}{a_{,}^3} b_{3,2} - \frac{3 a}{8 a_{,}^2} b_{3,3} \right\} e e_{,} \cos (n t - 3 n_{,} t + \varepsilon - 3 \varepsilon_{,} + \varpi + \varpi_{,}) \quad [53] \quad [49]$$

$$+ m_{,} \left\{ \frac{15}{4} \frac{a}{a_{,}^2} b_{3,2} - \frac{3 a^2}{a_{,}^3} b_{3,3} - \frac{a}{2 a_{,}^2} b_{3,4} \right\} e e_{,} \cos (2 n t - 4 n_{,} t + 2 \varepsilon - 4 \varepsilon_{,} + \varpi + \varpi_{,}) \quad [54] \quad [50]$$

$$+ m_{,} \left\{ \frac{39}{8} \frac{a}{a_{,}^2} b_{3,3} - \frac{4 a^2}{a_{,}^3} b_{3,4} - \frac{5}{8} \frac{a}{a_{,}^2} b_{3,5} \right\} e e_{,} \cos (3 n t - 5 n_{,} t + 3 \varepsilon - 5 \varepsilon_{,} - \varpi + \varpi_{,}) \quad [55] \quad [51]$$

$$+ m_{,} \left\{ 6 \frac{a}{a_{,}^2} b_{3,4} - \frac{5 a^2}{a_{,3}} b_{3,5} - \frac{3}{4} \frac{a}{a_{,}^2} b_{3,6} \right\} e e_{,} \cos (4 n t - 6 n_{,} t + 4 \varepsilon - 6 \varepsilon_{,} - \varpi + \varpi_{,}) \quad [56] \quad [52]$$

$$+ m_{,} \left\{ - \frac{1}{a_{,}} b_{3,0} + \frac{3}{8} \frac{a}{a_{,}^2} b_{3,1} \right\} e_{,}^2 \cos (2 n_{,} t + 2 \varepsilon_{,} - 2 \varpi_{,}) \quad [57] \quad [26]$$

$$+ m_{,} \left\{ \frac{a}{8 a_{,}^2} - \frac{a}{8 a_{,}^2} b_{3,0} - \frac{a}{16 a_{,}^2} b_{3,2} \right\} e_{,}^2 \cos (n t + n_{,} t + \varepsilon + \varepsilon_{,} - 2 \varpi_{,}) \quad [58] \quad [25]$$

$$+ m_{,} \left\{ - \frac{5}{8} \frac{a}{a_{,}^2} b_{3,1} + \frac{1}{2 a_{,}} b_{3,2} \right\} e_{,}^2 \cos (2 n t + 2 \varepsilon - 2 \varpi_{,}) \quad [59] \quad [24]$$

$$+ m_{,} \left\{ - \frac{19}{16} \frac{a}{a_{,}^2} b_{3,2} + \frac{1}{a_{,}} b_{3,3} + \frac{a}{16 a_{,}^2} b_{3,4} \right\} e_{,}^2 \cos (3 n t - n_{,} t + 3 \varepsilon - \varepsilon_{,} - 2 \varpi_{,}) \quad [60] \quad [32]$$

$$+ m_{,} \left\{ - \frac{7}{4} \frac{a}{a_{,}^2} b_{3,3} + \frac{3}{2 a_{,}} b_{3,4} + \frac{a}{8 a_{,}^2} b_{3,5} \right\} e_{,} \cos (4 n t - 2 n_{,} t + 4 \varepsilon - 2 \varepsilon_{,} - 2 \varpi_{,}) \quad [61] \quad [33]$$

$$+ m_{,} \left\{ - \frac{37}{16} \frac{a}{a_{,}^2} b_{3,4} + \frac{2}{a_{,}} b_{3,5} + \frac{3}{16} \frac{a}{a_{,}^2} b_{3,6} \right\} e_{,}^2 \cos 5 n t - 3 n_{,} t + 5 \varepsilon - 3 \varepsilon_{,} - 2 \varpi_{,}) \quad [62] \quad [34]$$

$$+ m_{,} \left\{ \frac{27}{8} \frac{a}{a_{,}^2} - \frac{3}{8} \frac{a}{a_{,}^2} b_{3,0} - \frac{1}{a_{,}} b_{3,1} + \frac{17}{16} \frac{a}{a_{,}^2} b_{3,2} \right\} e_{,}^2 \cos (n t - 3 n_{,} t + \varepsilon - 3 \varepsilon_{,} + 2 \varpi_{,}) \quad [63] \quad [27]$$

$$+ m_{,} \left\{ - \frac{a}{4 a_{,}^2} b_{3,1} - \frac{3}{2 a_{,}} b_{3,2} + \frac{13}{8} \frac{a}{a_{,}^2} b_{3,3} \right\} e_{,}^2 \cos (2 n t - 4 n_{,} t + 2 \varepsilon - 4 \varepsilon_{,} + 2 \varpi_{,}) \quad [64] \quad [28]$$

$$+ m_{,} \left\{ - \frac{5}{16} \frac{a}{a_{,}^2} b_{3,2} - \frac{2}{a_{,}} b_{3,3} + \frac{35}{16} \frac{a}{a_{,}^2} b_{3,4} \right\} e_{,}^2 \cos (3 n t - 5 n_{,} t + 3 \varepsilon - 5 \varepsilon_{,} + 2 \varpi_{,}) \quad [65] \quad [29]$$

$$+ m_{,} \left\{ - \frac{3}{8} \frac{a}{a_{,}^2} b_{3,3} - \frac{5}{2 a_{,}} b_{3,4} + \frac{11}{4} \frac{a}{a_{,}^2} b_{3,5} \right\} e_{,}^2 \cos (4 n t - 6 n_{,} t + 4 \varepsilon - 6 \varepsilon_{,} + 2 \varpi) \quad [66] \quad [30]$$

$$+ m_{\prime}\left\{-\frac{7}{16}\frac{a}{a_{\prime}^{2}}b_{3,4}-\frac{3}{a_{\prime}}b_{3,5}+\frac{53}{16}\frac{a}{a_{\prime}^{2}}b_{3,6}\right\}e_{\prime}^{2}\cos(5nt-7n_{\prime}t+5\varepsilon-7\varepsilon_{\prime}+2\varpi_{\prime}) \quad [67] \quad [31]$$

Development of R.

$$+ m_{\prime}\left\{\frac{a}{a_{\prime}^{2}}-\frac{a}{a_{\prime}^{2}}b_{3,0}\right\}\sin^{2}\frac{\iota_{\prime}}{2}(nt+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-2\nu_{\prime}) \quad [68]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,1}\sin^{2}\frac{\iota_{\prime}}{2}\cos(2n_{\prime}t+2\varepsilon_{\prime}-2\nu_{\prime}) \quad [69]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,1}\sin^{2}\frac{\iota_{\prime}}{2}\cos(2nt+2\varepsilon-2\nu_{\prime}) \quad [70]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,2}\sin^{2}\frac{\iota_{\prime}}{2}\cos(nt+3n_{\prime}t+\varepsilon-3\varepsilon_{\prime}+2\nu_{\prime}) \quad [71]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,2}\sin^{2}\frac{\iota_{\prime}}{2}\cos(3nt-n_{\prime}t+3\varepsilon-\varepsilon_{\prime}-2\nu_{\prime}) \quad [72]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,3}\sin^{2}\frac{\iota_{\prime}}{2}\cos(2nt-4n_{\prime}t+2\varepsilon-4\varepsilon_{\prime}+2\nu_{\prime}) \quad [73]$$

$$-\frac{m_{\prime}}{2}\frac{a}{a_{\prime}^{2}}b_{3,3}\sin^{2}\frac{\iota_{\prime}}{2}\cos(4nt-2n_{\prime}t+4\varepsilon-2\varepsilon_{\prime}-2\nu_{\prime}) \quad [74]$$

In the lunar theory, the small value of the quantity $\frac{a}{a_{\prime}}$ makes it desirable to ordain the results according to powers of this quantity. Transforming therefore the preceding expression for R by means of the equations given in the former part of this paper, Phil. Trans. for 1830, p. 346, neglecting terms multiplied by $\frac{a^{4}}{a_{\prime}^{5}}$, and supposing $\iota_{\prime}=0$,

$$R=m_{\prime}\left\{-\frac{1}{a_{\prime}}\left(1+\frac{a^{2}}{4a_{\prime}^{2}}\right)+\frac{3}{2}\left(\sin^{2}\frac{\iota}{2}-\frac{(e^{2}+e_{\prime}^{2})}{4}\right)\frac{a^{2}}{a_{\prime}^{3}}\right\} \quad [0]$$

$$+m_{\prime}\left\{\left(\cos^{2}\frac{\iota}{2}-\frac{(e^{2}+e_{\prime}^{2})}{2}\right)\frac{a}{a_{\prime}^{2}}-\frac{a}{a_{\prime}^{2}}\left(1+\frac{3}{8}\frac{a}{a_{\prime}^{2}}\right)+\sin^{2}\frac{\iota}{2}\frac{a}{a_{\prime}^{2}}\left(1+\frac{33}{8}\frac{a^{2}}{a_{\prime}^{2}}\right)\right.$$
$$\left.+\frac{(e^{2}+e_{\prime}^{2})}{2}\frac{a}{a_{\prime}^{2}}\left(1-\frac{3}{2}\frac{a^{2}}{a_{\prime}^{2}}\right)\right\}\cos(nt-n_{\prime}t+\varepsilon-\varepsilon_{\prime}) \quad [1]$$

$$+m_{\prime}\left\{-\frac{3}{4}\frac{a^{2}}{a_{\prime}^{3}}+\frac{3}{2}\sin^{2}\frac{\iota}{2}\frac{a^{2}}{a_{\prime}^{3}}+\frac{15}{8}(e^{2}+e_{\prime}^{2})\frac{a^{2}}{a_{\prime}^{3}}\right\}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}) \quad [2]$$

$$+m_{\prime}\left\{-\frac{5}{8}\frac{a^{3}}{a_{\prime}^{4}}+\frac{15}{8}\sin^{2}\frac{\iota}{2}\frac{a^{3}}{a_{\prime}^{4}}+\frac{15}{4}(e^{2}+e_{\prime}^{2})\frac{a^{3}}{a_{\prime}^{4}}\right\}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}) \quad [3]$$

Development of R according to powers of $\frac{a}{a_{\prime}}$.

$$+ m_{\prime} \frac{15}{16} \frac{a^3}{a_{\prime}^4} e \cos (n_{\prime} t + \varepsilon_{\prime} - \varpi) \qquad [6]$$

$$+ \frac{m_{\prime}}{2} \frac{a^2}{a_{\prime}^3} e \cos (n t + \varepsilon - \varpi) \qquad [7]$$

$$+ m_{\prime} \frac{3}{16} \frac{a^3}{a_{\prime}^4} e \cos (2 n t - n_{\prime} t + 2 \varepsilon - \varepsilon_{\prime} - \varpi) \qquad [8]$$

$$- m_{\prime} \frac{3}{4} \frac{a^2}{a_{\prime}^3} e \cos (3 n t - 2 n_{\prime} t + 3 \varepsilon - 2 \varepsilon_{\prime} - \varpi) \qquad [9]$$

$$- m_{\prime} \frac{15}{16} \frac{a^3}{a_{\prime}^4} e \cos (4 n t - 3 n_{\prime} t + 4 \varepsilon - 3 \varepsilon_{\prime} - \varpi) \qquad [10]$$

$$+ m_{\prime} \frac{9}{4} \frac{a^2}{a_{\prime}^3} e \cos (n t - 2 n_{\prime} t + \varepsilon - 2 \varepsilon_{\prime} + \varpi) \qquad [12]$$

$$+ m_{\prime} \frac{45}{16} \frac{a^3}{a_{\prime}^4} e \cos (2 n t - 3 n_{\prime} t + 2 \varepsilon - 3 \varepsilon_{\prime} + \varpi) \qquad [13]$$

$$- m_{\prime} \left\{ \frac{1}{a_{\prime}} + \frac{3}{4} \frac{a^2}{a_{\prime}^3} \right\} e_{\prime} \cos (n_{\prime} t + \varepsilon_{\prime} - \varpi_{\prime}) \qquad [15]$$

$$- m_{\prime} \frac{3}{8} \frac{a^3}{a_{\prime}^4} e_{\prime} \cos (n t + \varepsilon_{\prime} - \varpi_{\prime}) \qquad [16]$$

$$+ m_{\prime} \frac{3}{8} \frac{a^2}{a_{\prime}^3} e_{\prime} \cos (2 n t - n_{\prime} t + 2 \varepsilon - \varepsilon_{\prime} - \varpi_{\prime}) \qquad [17]$$

$$+ m_{\prime} \frac{5}{8} \frac{a^3}{a_{\prime}^4} e_{\prime} \cos (3 n t - 2 n_{\prime} t + 3 \varepsilon - 2 \varepsilon_{\prime} - \varpi_{\prime}) \qquad [18]$$

$$- m_{\prime} \frac{9}{8} \frac{a^3}{a_{\prime}^4} e_{\prime} \cos (n t - 2 n_{\prime} t + \varepsilon - 2 \varepsilon_{\prime} + \varpi_{\prime}) \qquad [20]$$

$$- m_{\prime} \frac{21}{8} \frac{a^2}{a_{\prime}^3} e_{\prime} \cos (2 n t - 3 n_{\prime} t + 2 \varepsilon - 3 \varepsilon_{\prime} + \varpi_{\prime}) \qquad [21]$$

$$- m_{\prime} \frac{25}{8} \frac{a^3}{a_{\prime}^4} e_{\prime} \cos (3 n t - 4 n_{\prime} t + 3 \varepsilon - 4 \varepsilon_{\prime} + \varpi_{\prime}) \qquad [22]$$

$$- m_{\prime} \frac{15}{8} \frac{a^2}{a_{\prime}^3} e^2 \cos (2 n_{\prime} t + 2 \varepsilon_{\prime} - 2 \varpi) \qquad [24]$$

$$- m_{\prime} \frac{33}{64} \frac{a^3}{a_{\prime}^4} e^2 \cos (n t + n_{\prime} t + \varepsilon + \varepsilon_{\prime} - 2 \varpi) \qquad [25]$$

$$+ \frac{m_{\prime}}{8} \frac{a^2}{a_{\prime}^3} e^2 \cos (2 n t + 2 \varepsilon - 2 \varpi) \qquad [26]$$

$$+ m_{\prime} \frac{9}{64} \frac{a^3}{a_{\prime}^4} e^2 \cos (3 n t - n_{\prime} t + 3 \varepsilon - \varepsilon_{\prime} - 2 \varpi) \qquad [27]$$

Development of R according to powers of $\frac{a}{a_{\prime}}$.

$$-m_{\prime}\frac{3}{4}\frac{a^2}{a_{\prime}^3}e^2\cos(4nt-2n_{\prime}t+4\varepsilon-2\varepsilon_{\prime}-2\varpi)$$ [28]

$$-m_{\prime}\frac{75}{64}\frac{a^3}{a_{\prime}^4}e^2\cos(5nt-3n_{\prime}t+5\varepsilon-3\varepsilon_{\prime}-2\varpi)$$ [29]

$$-m_{\prime}\frac{285}{64}\frac{a^3}{a_{\prime}^4}e^2\cos(nt-3n_{\prime}t+\varepsilon-3\varepsilon_{\prime}+2\varpi)$$ [32]

$$+m_{\prime}\frac{3}{4}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(nt-n_{\prime}t+\varepsilon-\varepsilon_{\prime}-\varpi+\varpi_{\prime})$$ [35]

$$+m_{\prime}\frac{9}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}-\varpi+\varpi_{\prime})$$ [36]

$$-m_{\prime}\frac{21}{8}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}-\varpi+\varpi_{\prime})$$ [37]

$$+m_{\prime}\frac{15}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(\varpi-\varpi_{\prime})$$ [41]

$$-m_{\prime}\frac{9}{8}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(nt-n_{\prime}t+\varepsilon-\varepsilon_{\prime}+\varpi-\varpi_{\prime})$$ [42]

$$-m_{\prime}\frac{45}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}+\varpi-\varpi_{\prime})$$ [43]

$$+m_{\prime}\frac{45}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(2n_{\prime}t+2\varepsilon_{\prime}-\varpi-\varpi_{\prime})$$ [46]

$$+m_{\prime}\frac{3}{4}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(nt+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-\varpi-\varpi_{\prime})$$ [47]

$$+m_{\prime}\frac{3}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(2nt+2\varepsilon-\varpi-\varpi_{\prime})$$ [48]

$$+m_{\prime}\frac{3}{8}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(3nt-n_{\prime}t+3\varepsilon-\varepsilon_{\prime}-\varpi-\varpi_{\prime})$$ [49]

$$+m_{\prime}\frac{15}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(4nt-2n_{\prime}t+4\varepsilon-2\varepsilon_{\prime}-\varpi-\varpi_{\prime})$$ [50]

$$+m_{\prime}\frac{63}{8}\frac{a^2}{a_{\prime}^3}ee_{\prime}\cos(nt-3n_{\prime}t+\varepsilon-3\varepsilon_{\prime}+\varpi+\varpi_{\prime})$$ [53]

$$+m_{\prime}\frac{225}{16}\frac{a^3}{a_{\prime}^4}ee_{\prime}\cos(2nt-4n_{\prime}t+2\varepsilon-4\varepsilon_{\prime}+\varpi+\varpi_{\prime})$$ [54]

Development of R according to powers of $\frac{a}{a_{,}}$.

$$+ m_{,}\left\{\frac{1}{a_{,}} - \frac{9}{8}\frac{a^2}{a_{,}^3}\right\} e_{,}^2 \cos(2 n_{,} t + 2 \varepsilon_{,} - 2 \varpi_{,}) \qquad [57]$$

$$- m_{,}\frac{33}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(n t + n_{,} t + \varepsilon + \varepsilon_{,} - 2 \varpi_{,}) \qquad [58]$$

$$- m_{,}\frac{5}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(3 n t - n_{,} t + 3 \varepsilon - \varepsilon_{,} - 2 \varpi_{,}) \qquad [60]$$

$$- m_{,}\frac{159}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(n t - 3 n_{,} t + \varepsilon - 3 \varepsilon_{,} + 2 \varpi_{,}) \qquad [63]$$

$$- m_{,}\frac{51}{8}\frac{a^2}{a_{,}^3} e_{,}^2 \cos(2 n t - 4 n_{,} t + 2 \varepsilon - 4 \varepsilon_{,} + 2 \varpi_{,}) \qquad [64]$$

$$- m_{,}\frac{635}{64}\frac{a^3}{a_{,}^4} e_{,}^2 \cos(3 n t - 5 n_{,} t + 3 \varepsilon - 5 \varepsilon_{,} + 2 \varpi_{,}) \qquad [65]$$

$$- m_{,}\frac{9}{4}\frac{a^3}{a_{,}^4}\sin^2\frac{\iota}{2} \cos(n t + n_{,} t + \varepsilon + \varepsilon_{,} - 2 \nu) \qquad [75]$$

$$- m_{,}\frac{3}{2}\frac{a^2}{a_{,}^3}\sin^2\frac{\iota}{2} \cos(2 n t + 2 \varepsilon - 2 \nu) \qquad [76]$$

$$- m_{,}\frac{3}{2}\frac{a^2}{a_{,}^3}\sin^2\frac{\iota}{2} \cos(2 n_{,} t + 2 \varepsilon_{,} - 2 \nu) \qquad [77]$$

$$- m_{,}\frac{15}{8}\frac{a^3}{a_{,}^4}\sin^2\frac{\iota}{2} \cos(3 n t - n_{,} t + 3 \varepsilon - \varepsilon_{,} - 2 \nu) \qquad [78]$$

$$- m_{,}\frac{15}{8}\frac{a^3}{a_{,}^4}\sin^2\frac{\iota}{2} \cos(n t - 3 n_{,} t + \varepsilon - 3 \varepsilon_{,} + 2 \nu) \qquad [79]$$

If according to the notation of M. Damoiseau, (Théorie Lunaire, p. 547, Mémoires des Savans Étrangers,) $n t - n_{,} t + \varepsilon - \varepsilon_{,} = t$, and x and z be put for the mean anomalies of m and $m_{,}$ respectively and y for the distance of the planet m from its node, or, what is the same in the Lunar Theory, the distance of the moon from her node reckoned on the ecliptic ($\iota_{,} = 0$),

$$R = m_{,}\left\{-\frac{1}{a_{,}}\left(1 + \frac{a^2}{4 a_{,}^2}\right) + \frac{3}{2}\left(\sin^2\frac{\iota}{2} - \frac{(e^2 + e_{,}^2)}{4}\right)\frac{a^2}{a_{,}^3}\right. \quad [0]$$

$$+ \left\{\left(\cos^2\frac{\iota_{,}}{2} - \frac{(e^2 + e_{,}^2)}{2}\right)\frac{a}{a_{,}^2} - \frac{a}{a_{,}^2}\left(1 + \frac{3}{8}\frac{a^2}{a_{,}^2}\right) + \sin^2\frac{\iota}{2}\frac{a}{a_{,}^2}\left(1 + \frac{33}{8}\frac{a^2}{a_{,}^2}\right)\right.$$

$$\left. + \frac{(e^2 + e_{,}^2)}{2}\frac{a}{a_{,}^2}\left(1 - \frac{3}{2}\frac{a^2}{a_{,}^2}\right)\right\}\cos t + \left\{-\frac{3}{4}\frac{a^2}{a_{,}^3} + \frac{3}{2}\sin^2\frac{\iota}{2}\frac{a^2}{a_{,}^3} + \frac{15}{8}(e^2 + e_{,}^2)\frac{a^2}{a_{,}^3}\right\}\cos 2 t$$

[1] [2]

Development of R according to powers of $\frac{a}{a'}$.

$$+\left\{-\frac{5}{8}\frac{a^3}{a_{,}^4}+\frac{15}{8}\sin^2\frac{\iota}{2}\,\frac{a^3}{a_{,}^4}+\frac{15}{4}\,(e^2+e_{,}^2)\,\frac{a^3}{a_{,}^4}\right\}\cos 3\,t+\frac{15}{16}\frac{a^3}{a_{,}^4}\cos\,(t-x)$$

[3] [6]

$$+\frac{a^3}{2\,a_{,}^3}\,e\cos x+\frac{3}{16}\frac{a^3}{a_{,}^4}\,e\cos t+x)-\frac{3}{4}\frac{a^2}{a_{,}^3}\,e\cos\,(2\,t+x)-\frac{15}{16}\frac{a^3}{a_{,}^4}\,e\cos\,(3\,t+x)$$

[7] [8] [9] [10]

$$+\frac{9}{4}\frac{a^2}{a_{,}^3}\,e\cos\,(2\,t-x)+\frac{45}{16}\frac{a^3}{a_{,}^4}\,e\cos\,(3\,t-x)-\left\{\frac{1}{a_{,}}+\frac{3}{4}\frac{a^2}{a_{,}^3}\right\}e_{,}\cos z$$

[12] [13] [15]

$$-\frac{3}{8}\frac{a^3}{a_{,}^4}\,e_{,}\cos\,(t+z)+\frac{3}{8}\frac{a^2}{a_{,}^3}\,e_{,}\cos\,(2\,t+z)+\frac{5}{8}\frac{a_{,}^3}{a_{,}^4}\,e_{,}\cos\,(3\,t+z)$$

[16] [17] [18]

$$-\frac{9}{8}\frac{a^3}{a_{,}^4}\,e_{,}\cos\,(t-z)-\frac{21}{8}\frac{a^2}{a_{,}^3}\,e_{,}\cos\,(2\,t-z)-\frac{25}{8}\frac{a^3}{a_{,}^4}\,e_{,}\cos\,(3\,t-z)$$

[20] [21] [22]

$$-\frac{15}{8}\frac{a^2}{a_{,}^3}\,e^2\cos\,(2\,t-2\,x)-\frac{33}{64}\frac{a^3}{a_{,}^4}\,e^2\cos\,(t-2\,x)+\frac{a^2}{8\,a_{,}^3}\,e^2\cos 2\,x$$

[24] [25] [26]

$$+\frac{9}{64}\frac{a^3}{a_{,}^4}\,e^2\cos\,(t+2\,x)-\frac{3}{4}\frac{a^2}{a_{,}^3}\,e^2\cos\,(2\,t+2\,x)-\frac{75}{64}\frac{a^3}{a_{,}^4}\,e^2\cos\,(3\,t+2\,x)$$

[27] [28] [29]

$$-\frac{285}{64}\frac{a^3}{a_{,}^4}\,e^2\cos\,(3\,t-2\,x)+\frac{3}{4}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(x-z)+\frac{9}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(t-z+x)$$

[32] [35] [36]

$$-\frac{27}{8}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(2\,t-z+x)+\frac{15}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(t+z-x)-\frac{9}{8}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(2\,t+z-x)$$

[37] [41] [42]

$$-\frac{45}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(3\,t+z-x)+\frac{45}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(t-z-x)+\frac{3}{4}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(x+z)$$

[43] [46] [47]

$$+\frac{3}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(t+z+x)+\frac{3}{8}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(2\,t+z+x)+\frac{15}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(3\,t+z+x)$$

[48] [49] [50]

$$+\frac{63}{8}\frac{a^2}{a_{,}^3}\,e\,e_{,}\cos\,(2\,t-z-x)-\frac{225}{16}\frac{a^3}{a_{,}^4}\,e\,e_{,}\cos\,(3\,t-z-x)+\left\{\frac{1}{a_{,}}-\frac{9}{8}\frac{a^2}{a_{,}^3}\right\}e_{,}^2\cos 2\,z$$

[53] [54] [57]

$$-\frac{33}{64}\frac{a^3}{a_{,}^4}\,e_{,}^2\cos\,(t+2\,z)-\frac{5}{64}\frac{a^3}{a_{,}^4}\,e_{,}^2\cos\,(3\,t+2\,z)-\frac{159}{64}\frac{a^3}{a_{,}^4}\,e_{,}^2\cos\,(t-2\,z)$$

[58] [60] [63]

$$-\frac{51}{8}\frac{a^2}{a_{,}^3}\,e_{,}^2\,(2\,t-2\,z)-\frac{635}{64}\frac{a^3}{a_{,}^4}\,e_{,}^2\cos\,(3\,t-2\,z)-\frac{9}{4}\frac{a^3}{a_{,}^4}\sin^2\frac{\iota}{2}\cos\,(t-2\,y)$$

[64] [65] [75]

$$- \frac{3}{2} \frac{a^2}{a_i^3} \sin^2 \frac{\iota}{2} \cos 2y - \frac{3}{2} \frac{a^2}{a_i^3} \sin^2 \frac{\iota}{2} \cos (2t - 2y) - \frac{15}{8} \frac{a^3}{a_i^4} \cos (t + 2y)$$

[76] [77] [78]

$$- \frac{15}{8} \frac{a^3}{a_i^4} \sin^2 \frac{\iota}{2} \cos (3t - 2y) \Big\}$$

[79]

Let $\frac{a}{r} = 1 + e \cos \left(n (1 + k) t + \varepsilon - \varpi \right) + e^2 (1 + r_{48}) \cos \left(2n (1 + k_2) t + 2\varepsilon - 2\varpi \right)$

$$+ e_i^2 r_{59} \cos \left(2n (1 + k_{12}) t + 2\varepsilon - 2\varpi_i \right) + r_0 + r_1 \cos (nt + n_i t + \varepsilon - \varepsilon_i)$$

$$+ r_2 \cos (2nt - 2n_i t + 2\varepsilon - 2\varepsilon_i) + \&c. + e r_6 \cos (n_i t + \varepsilon_i - \varpi) + \&c.$$

$$\frac{d^2 r^2}{2\, d t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int d R + r \left(\frac{d R}{d r} \right) = 0$$

Let $\delta . \frac{1}{r}$ denote that part of $\frac{1}{r}$ which depends on the first power of the disturbing force. It is more simple to obtain $\delta . \frac{1}{r}$ from the above differential equation than δr; and the circumstance that the elliptic value of r^3 does not contain any term $e^2 \cos (2nt + 2\varepsilon - 2\varpi)$, gives an additional facility.

$$- \frac{d^2 . r^3 \delta . \frac{1}{r}}{d t^2} - \mu \delta . \frac{1}{r} + 2 \int d R + r \left(\frac{d R}{d r} \right) = 0$$

When the disturbing force is neglected

$$r^3 = a^3 \left\{ 1 + 3e^2 \left(1 + \frac{e^2}{12} \right) - 3e \left(1 + \frac{3}{8} e^2 \right) \cos (nt + \varepsilon - \varpi) - \frac{3}{8} e^4 \cos (2nt + 2\varepsilon - 2\varpi) \right.$$

$$\left. + \frac{e^3}{8} \cos (3nt + 3\varepsilon - 3\varpi) + \frac{e^4}{8} \cos (4nt + 4\varepsilon - 4\varpi) \right\}$$

Integrating the above differential equation by the method of indeterminate co-efficients, q_n being the co-efficient of the nth argument in the development of $2 \int d R + r \left(\frac{d R}{d r} \right)$,

$$- r_0 + \frac{m_i}{\mu} a q_0 = 0$$

$$\frac{(n - n_i)^2}{n^2} \left\{ (1 + 3e^2) r_1 - \frac{3e^2}{2} (r_6 + r_8) \right\} - r_1 + \frac{m_i}{\mu} a q_1 = 0$$

$$\frac{(2n - 2n_i)^2}{n^2} \left\{ (1 + 3e^2) r_2 - \frac{3e^2}{2} (r_9 + r_{12}) \right\} - r_2 + \frac{m_i}{\mu} a q_2 = 0$$

Equations which serve to determine the coefficients of the inequalities of the reciprocal of the radius vector.

$$\frac{(3n-3n_{,})^2}{n^2}\left\{(1+3e^2)r_3-\frac{3e^2}{2}(r_{10}+r_{13})\right\}-r_3+\frac{m_{,}}{\mu}aq_3=0$$

$$\frac{(4n-4n_{,})}{n^2}\left\{(1+3e^2)r_4-\frac{3e^2}{2}(r_{11}+r_{14})\right\}-r_4+\frac{m_{,}}{\mu}aq_4=0$$

$$\frac{(5n-5n_{,})^2}{n^2}(1+3e^2)r_5-r_5+\frac{m_{,}}{\mu}aq_5=0$$

$$\frac{n^2}{n_{,}^2}\left\{r_6-\frac{3}{2}r_1\right\}-r_6+\frac{m_{,}}{\mu}aq_6=0$$

$$*\frac{(2n-n_{,})^2}{n^2}\left\{r_8-\frac{3}{2}r_1\right\}-r_8+\frac{m_{,}}{\mu}aq_8=0$$

$$\frac{(3n-2n_{,})^2}{n^2}\left\{r_9-\frac{3}{2}r_2\right\}-r_9+\frac{m_{,}}{\mu}aq_9=0$$

$$\frac{(4n-3n_{,})^2}{n^2}\left\{r_{10}-\frac{3}{2}r_3\right\}-r_{10}+\frac{m_{,}}{\mu}aq_{10}=0$$

$$\frac{(5n-4n_{,})^2}{n^2}\left\{r_{11}-\frac{3}{2}r_4\right\}-r_{11}+\frac{m_{,}}{\mu}aq_{11}=0$$

$$\frac{(n-2n_{,})^2}{n^2}\left\{r_{12}-\frac{3}{2}r_2\right\}-r_{12}+\frac{m_{,}}{\mu}aq_{12}=0$$

$$\frac{(2n-3n_{,})^2}{n^2}\left\{r_{13}-\frac{3}{2}r_3\right\}-r_{13}+\frac{m_{,}}{\mu}aq_{13}=0$$

$$\frac{(3n-4n_{,})^2}{n^2}\left\{r_{14}-\frac{3}{2}r_4\right\}-r_{14}+\frac{m_{,}}{\mu}aq_{14}=0$$

$$\frac{n_{,}^2}{n^2}r_{15}-r_{15}+\frac{m_{,}}{\mu}aq_{15}=0$$

$$\frac{(2n-n_{,})^2}{n^2}r_{17}-r_{17}+\frac{m_{,}}{\mu}aq_{17}=0$$

$$\frac{(3n-2n_{,})^2}{n^2}r_{18}-r_{18}+\frac{m_{,}}{\mu}aq_{18}=0$$

$$\frac{(4n-3n_{,})^2}{n^2}r_{19}-r_{19}+\frac{m_{,}}{\mu}aq_{19}=0$$

$$\frac{(n-3n_{,})^2}{n^2}r_{20}-r_{20}+\frac{m_{,}}{\mu}aq_{20}-0$$

$$\frac{(2n-3n_{,})^2}{n^2}r_{21}-r_{21}+\frac{m_{,}}{\mu}aq_{21}=0$$

$$\frac{(3n-4n_{,})^2}{n^2}r_{22}-r_{22}+\frac{m_{,}}{\mu}aq_{22}=0$$

* For the determination of the quantity k, see p. 50.

Equations which serve to determine the coefficients of the inequalities of the reciprocal of the radius vector.

$$\frac{(4n - 5n_{\prime})^2}{n^2} r_{23} - r_{23} + \frac{m_{\prime}}{\mu} a q_{23} = 0$$

$$\frac{4n_{\prime}^2}{n^2}\left\{r_{24} - \frac{3}{2} r_{12}\right\} - r_{24} + \frac{m_{\prime}}{\mu} a q_{24} = 0$$

$$\frac{(n + n_{\prime})^2}{n^2}\left\{r_{25} - \frac{3}{2} r_{6}\right\} - r_{25} + \frac{m_{\prime}}{\mu} a q_{25} = 0$$

$$\frac{(3n - n_{\prime})^2}{n^2}\left\{r_{27} - \frac{3}{2} r_{8}\right\} - r_{27} + \frac{m_{\prime}}{\mu} a q_{27} = 0$$

$$\frac{(4n - 2n_{\prime})^2}{n^2}\left\{r_{28} - \frac{3}{2} r_{9}\right\} - r_{28} + \frac{m_{\prime}}{\mu} a q_{28} = 0$$

$$\frac{(5n - 3n_{\prime})^2}{n^2}\left\{r_{29} - \frac{3}{2} r_{10}\right\} - r_{29} + \frac{m_{\prime}}{\mu} a q_{29} = 0$$

$$\frac{(6n - 4n_{\prime})^2}{n^2}\left\{r_{30} - \frac{3}{2} r_{11}\right\} - r_{30} + \frac{m_{\prime}}{\mu} a q_{30} = 0$$

$$\frac{(7n - 5n_{\prime})^2}{n^2} r_{31} - r_{31} + \frac{m_{\prime}}{\mu} a q_{31} = 0$$

$$\frac{(n - 3n_{\prime})^2}{n^2}\left\{r_{32} - \frac{3}{2} r_{13}\right\} - r_{32} + \frac{m_{\prime}}{\mu} a q_{32} = 0$$

$$\frac{(2n - 4n_{\prime})^2}{n^2}\left\{r_{33} - \frac{3}{2} r_{14}\right\} - r_{33} + \frac{m_{\prime}}{\mu} a q_{33} = 0$$

$$\frac{(3n - 5n_{\prime})^2}{n^2} r_{34} - r_{34} + \frac{m_{\prime}}{\mu} a q_{34} = 0$$

$$\frac{(n - n_{\prime})^2}{n^2}\left\{r_{35} - \frac{3}{2} r_{15}\right\} - r_{35} + \frac{m_{\prime}}{\mu} a q_{35} = 0$$

$$\frac{(2n - 2n_{\prime})^2}{n^2}\left\{r_{36} - \frac{3}{2} r_{20}\right\} - r_{36} + \frac{m_{\prime}}{\mu} a q_{36} = 0$$

$$\frac{(3n - 3n_{\prime})^2}{n^2}\left\{r_{37} - \frac{3}{2} r_{21}\right\} - r_{37} + \frac{m_{\prime}}{\mu} a q_{37} = 0$$

$$\frac{(4n - 4n_{\prime})^2}{n^2}\left\{r_{38} - \frac{3}{2} r_{22}\right\} - r_{38} + \frac{m_{\prime}}{\mu} a q_{38} = 0$$

$$\frac{(5n - 5n_{\prime})^2}{n^2}\left\{r_{39} - \frac{3}{2} r_{23}\right\} - r_{39} + \frac{m_{\prime}}{\mu} a q_{39} = 0$$

$$\frac{(6n - 6n_{\prime})^2}{n^2} r_{40} - r_{40} + \frac{m_{\prime}}{\mu} a q_{40} = 0$$

$$- r_{41} + \frac{m_{\prime}}{\mu} a q_{41} = 0$$

$$\frac{(n - n_{\prime})^2}{n^2}\left\{r_{42} - \frac{3}{2} r_{17}\right\} - r_{42} + \frac{m_{\prime}}{\mu} a q_{42} = 0$$

Equations which serve to determine the coefficients of the inequalities of the reciprocal of the radius vector.

$$\frac{(2n-2n_{\prime})^2}{n^2}\left\{r_{43}-\frac{3}{2}r_{18}\right\}-r_{43}+\frac{m_{\prime}}{\mu}a\,q_{43}=0$$

$$\frac{(3n-3n_{\prime})^2}{n^2}\left\{r_{44}-\frac{3}{2}r_{19}\right\}-r_{44}+\frac{m_{\prime}}{\mu}a\,q_{44}=0$$

$$\frac{(4n-4n_{\prime})^2}{n^2}\left\{r_{45}-r_{45}+\frac{m_{\prime}}{\mu}a\,q_{45}=0\right.$$

$$\frac{4n_{\prime}^2}{n^2}\left\{r_{46}-\frac{3}{2}r_{20}\right\}-r_{46}+\frac{m_{\prime}}{\mu}a\,q_{46}=0$$

$$\frac{(n+n_{\prime})^2}{n^2}\left\{r_{47}-\frac{3}{2}r_{15}\right\}-r_{47}+\frac{m_{\prime}}{\mu}a\,q_{47}=0$$

$$\frac{(3n-3n_{\prime})^2}{n^2}\left\{r_{49}-\frac{3}{2}r_{17}\right\}-r_{49}+\frac{m_{\prime}}{\mu}a\,q_{45}=0$$

$$\frac{(4n-2n_{\prime})^2}{n^2}\left\{r_{50}-\frac{3}{2}r_{18}\right\}-r_{50}+\frac{m_{\prime}}{\mu}a\,q_{50}=0$$

$$\frac{(5n-5n_{\prime})^2}{n^2}\left\{r_{51}-\frac{3}{2}r_{19}\right\}-r_{51}+\frac{m_{\prime}}{\mu}a\,q_{51}=0$$

$$\frac{(6n-4n_{\prime})^2}{n^2}\left\{r_{52}-r_{52}+\frac{m_{\prime}}{\mu}a\,q_{52}=0\right.$$

$$\frac{(n-n_{\prime})^2}{n^2}\left\{r_{53}-\frac{3}{2}r_{21}\right\}-r_{53}+\frac{m_{\prime}}{\mu}a\,q_{53}=0$$

$$\frac{(2n-4n_{\prime})^2}{n}\left\{r_{54}-\frac{3}{2}r_{22}\right\}-r_{54}+\frac{m_{\prime}}{\mu}a\,q_{54}=0$$

$$\frac{(3n-5n_{\prime})^2}{n^2}\left\{r_{55}-\frac{3}{2}r_{23}\right\}-r_{55}+\frac{m_{\prime}}{\mu}a\,q_{55}=0$$

$$\frac{(4n-6n_{\prime})^2}{n^2}r_{56}-r_{56}+\frac{m_{\prime}}{\mu}a\,q_{56}=0$$

$$\frac{4n_{\prime}^2}{n^2}r_{57}-r_{57}+\frac{m_{\prime}}{\mu}a\,q_{57}=0$$

$$\frac{(m+n_{\prime})^2}{n^2}r_{58}-r_{58}+\frac{m_{\prime}}{\mu}a\,q_{58}=0$$

$$\frac{(3n-n_{\prime})^2}{n^2}r_{60}-r_{60}+\frac{m_{\prime}}{\mu}a\,q_{60}=0$$

$$\frac{(4n-2n_{\prime})^2}{n^2}r_{61}-r_{61}+\frac{m_{\prime}}{\mu}a\,q_{61}=0$$

$$\frac{(5n-3n_{\prime})^2}{n^2}r_{62}-r_{62}+\frac{m_{\prime}}{\mu}a\,q_{62}=0$$

$$\frac{(n-3n_{\prime})^2}{n^2}r_{65}-r_{63}+\frac{m_{\prime}}{\mu}a\,q_{62}=0$$

Equations which serve to determine the coefficients of the inequalities of the reciprocal of the radius vector.

$$\frac{(2n-4n_{\prime})^2}{n^2}r_{64}-r_{64}+\frac{m_{\prime}}{\mu}a\,q_{64}=0$$

$$\frac{(3n-5n_{\prime})^2}{n^2}r_{65}-r_{65}+\frac{m_{\prime}}{\mu}a\,q_{65}=0$$

$$\frac{(4n-6n_{\prime})^2}{n^2}r_{66}-r_{66}+\frac{m_{\prime}}{\mu}a\,q_{66}=0$$

$$\frac{(5n-7n_{\prime})^2}{n^2}r_{67}-r_{67}+\frac{m_{\prime}}{\mu}a\,q_{67}=0$$

$$\frac{(n+n_{\prime})^2}{n^2}r_{68}-r_{68}+\frac{m_{\prime}}{\mu}a\,q_{68}=0$$

In order to obtain the values of the coefficients of the inequalities from these equations when the cubes of the eccentricities are neglected, as has been the case throughout, the values of r_0, r_1, r_2, r_3, r_4, r_5, found from the first six equations by neglecting the terms multiplied by e^2, may be substituted in the succeeding equations, which will then serve to determine r_6, r_8, r_9, &c. and these values of r_6, r_8, r_9, &c. being substituted in the terms multiplied by e^2, of the equations which determine r_1, r_2, r_3, r_4, &c. more accurate values of those quantities may be obtained. All the other coefficients of which the general symbol is r with a numerical index at foot, may then be obtained in succession without any difficulty.

$$\frac{d\lambda}{dt}=\frac{h}{r^2}-\frac{1}{r^2}\int\frac{dR}{d\lambda}dt$$

Let r denote the elliptic value of r, then

$$\frac{d\lambda}{dt}=\frac{h}{\mathrm{r}^2}+\frac{2h}{\mathrm{r}}\,\delta\,.\,\frac{1}{r}-\frac{1}{\mathrm{r}^2}\int\frac{dR}{d\lambda}dt$$

$$\frac{a^2}{\mathrm{r}^2}=1+\frac{e^2}{2}+\frac{3}{8}e^4+2e\left(1+\frac{3e^2}{8}\right)\cos(nt-\varpi)+\frac{5e^2}{2}\left(1+\frac{2}{15}e^2\right)\cos(2nt-2\varpi)$$
$$+\frac{13}{4}e^3\cos(3nt-3\varpi)+\frac{103}{24}e^4\cos(4nt-4\varpi)$$

$$\frac{a}{\mathrm{r}}=1+e\left(1-\frac{e^2}{8}\right)\cos(nt-\varpi)+e^2\left(1-\frac{e^2}{3}\right)\cos(2nt-2\varpi)$$
$$+\frac{9}{8}e^3\cos(3nt-3\varpi)+\frac{4}{3}e^4\cos(4nt-4\varpi)$$

Let R_n denote the coefficient of the cosine in the development of R which corresponds to the number n multiplied by e or $e_{,}$, &c. Thus

$$R_6 = \left\{ -\frac{3\,a}{a_{,}^2} + \frac{3\,a}{2\,a_{,}^2} b_{3,0} - \frac{a^2}{2\,a_{,}^3} b_{3,1} - \frac{a}{4\,a_{,}^2} b_{3,2} \right\}$$ See p. 31

Since $\frac{d\,R}{d\,\lambda} = \frac{d\,R}{d\,\varepsilon}$, $\frac{d\,R}{d\,\varepsilon}$ being the differential of R with respect to ε, considering $\varepsilon - \varpi$ and $\varepsilon - \nu$ constant

$$\lambda = n \left\{ 1 + 2\,r_0 \right\} t + \varepsilon$$

$$+ \left\{ 2 \left\{ r_1 \left(1 - \frac{e^2}{2}\right) + \frac{e^2}{2}(r_6 + r_8) \right\} - \frac{m_{,}}{\mu} \left\{ \left(1 + \frac{e^2}{2}\right) \frac{a\,R_1\,n}{(n - n_{,})} - \frac{e^2}{n_{,}} a\,R_6\,n + \frac{e^2\,a\,R_8\,n}{(2\,n - n_{,})} \right\} \right\}$$

$$\frac{n}{(n - n_{,})} \sin(n\,t - n_{,}\,t + \varepsilon - \varepsilon_{,})$$

$$+ \left\{ 2 \left\{ r_2 \left(1 - \frac{e^2}{2}\right) + \frac{e^2}{2}(r_9 + r_{12}) \right\} - \frac{m_{,}}{\mu} \left\{ \left(1 + \frac{e^2}{2}\right) \frac{a\,R_2\,n}{(n - n_{,})} + \frac{2\,e^2\,a\,R_9\,n}{(3\,n - 2\,n_{,})} + \frac{2\,e^2\,a\,R_{12}\,n}{(n - 2\,n_{,})} \right\} \right\}$$

$$\frac{n}{(2\,n - 2\,n_{,})} \sin(2\,n\,t - 2\,n_{,}\,t + 2\,\varepsilon - 2\,\varepsilon_{,})$$

$$+ \left\{ 2 \left\{ r_3 \left(1 - \frac{e^2}{2}\right) + \frac{e^2}{2}(r_{10} + r_{13}) \right\} - \frac{m_{,}}{\mu} \left\{ \left(1 + \frac{e^2}{2}\right) \frac{a\,R_3\,n}{(n - n_{,})} + \frac{3\,e^2\,a\,R_{10}\,n}{(4\,n - 3\,n_{,})} + \frac{3\,e^2\,a\,R_{13}\,n}{(2\,n - 3\,n_{,})} \right\} \right\}$$

$$\frac{n}{(3\,n - 3\,n_{,})} \sin(3\,n\,t - 3\,n_{,}\,t + 3\,\varepsilon - 3\,\varepsilon_{,})$$

$$+ \left\{ 2 \left\{ r_4 \left(1 - \frac{e^2}{2}\right) + \frac{e^2}{2}(r_{11} + r_{14}) \right\} - \frac{m_{,}}{\mu} \left\{ \left(1 + \frac{e^2}{2}\right) \frac{a\,R_4\,n}{(n - n_{,})} + \frac{4\,e^2\,a\,R_{11}\,n}{(5\,n - 4\,n_{,})} + \frac{4\,e^2\,a\,R_{14}\,n}{(3\,n - 4\,n_{,})} \right\} \right\}$$

$$\frac{n}{(4\,n - 4\,n_{,})} \sin(4\,n\,t - 4\,n_{,}\,t + 4\,\varepsilon - 4\,\varepsilon_{,})$$

$$+ \left\{ 2 \left(r_6 + \frac{r_1}{2}\right) - \frac{m_{,}}{\mu} \left\{ -\frac{a\,R_6\,n}{n_{,}} + \frac{a\,R_1\,n}{(n - n_{,})} \right\} \right\} \frac{e}{n_{,}} \sin(n_{,}\,t + \varepsilon_{,} - \varpi)$$

$$+ \left\{ 2 \left(r_8 + \frac{r_1}{2}\right) - \frac{m_{,}}{\mu} \left\{ \frac{a\,R_8\,n}{(2\,n - n_{,})} + \frac{a\,R_1\,n}{(n - n_{,})} \right\} \right\} \frac{n\,e}{(n - n)} \sin(2\,n\,t - n_{,}\,t + 2\,\varepsilon - \varepsilon_{,} - \varpi)$$

$$+ \left\{ 2 \left(r_9 + \frac{r_2}{2}\right) - \frac{m_{,}}{\mu} \left\{ \frac{2\,a\,R_9\,n}{(3\,n - 2\,n_{,})} + \frac{a\,R_2\,n}{(n - n_{,})} \right\} \right\} \frac{n\,e}{(3\,n - 2\,n_{,})} \sin(3\,n\,t - 2\,n\,t + 3\,\varepsilon - 2\,\varepsilon_{,} - \varpi)$$

$$+ \left\{ 2 \left(r_{10} + \frac{r_3}{2}\right) - \frac{m_{,}}{\mu} \left\{ \frac{3\,a\,R_{10}\,n}{(4\,n - 3\,n_{,})} + \frac{a\,R_3\,n}{(n - n_{,})} \right\} \right\} \frac{n\,e}{(4\,n - 3\,n_{,})} \sin(4\,n\,t - 3\,n_{,}\,t + 4\,\varepsilon - 3\,\varepsilon_{,} - \varpi)$$

Expression for the longitude.

$$+\left\{2\left(r_{11}+\frac{r_4}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{11}\,n}{(5\,n-4\,n_{\prime})}+\frac{a\,R_4\,n}{(n-n_{\prime})}\right\}\right\}\frac{n\,e}{(5\,n-4\,n_{\prime})}\sin\left(5\,n\,t-4\,n_{\prime}\,t+5\,\varepsilon-4\,\varepsilon_{\prime}-\varpi\right)$$

$$+\left\{2\left(r_{12}+\frac{r_2}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{2\,a\,R_{12}\,n}{(n-2\,n_{\prime})}+\frac{a\,R_2\,n}{(n-n_{\prime})}\right\}\right\}\frac{n\,e}{(n-2\,n_{\prime})}\sin\left(n\,t-2\,n_{\prime}\,t+\varepsilon-2\,\varepsilon_{\prime}+\varpi\right)$$

$$+\left\{2\left(r_{13}+\frac{r_3}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{13}\,n}{(2\,n-3\,n_{\prime})}+\frac{a\,R_3\,n}{(n-n_{\prime})}\right\}\right\}\frac{n\,e}{(2\,n-3\,n_{\prime})}\sin\left(2\,n\,t-3\,n_{\prime}\,t+2\,\varepsilon-3\,\varepsilon_{\prime}+\varpi\right)$$

$$+\left\{2\left(r_{14}+\frac{r_4}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{14}\,n}{(3\,n-4\,n_{\prime})}+\frac{a\,R_4\,n}{(n-n_{\prime})}\right\}\right\}\frac{n\,e}{(3\,n-4\,n_{\prime})}\sin\left(3\,n\,t-4\,n_{\prime}\,t+3\,\varepsilon-4\,\varepsilon_{\prime}+\varpi\right)$$

$$+\,2\,r_{15}\,\frac{n\,e_{\prime}}{n_{\prime}}\sin\left(n_{\prime}\,t+\varepsilon_{\prime}-\varpi_{\prime}\right)$$

$$+\left\{2\,r_{17}-\frac{2\,m_{\prime}\,a\,R_{17}\,n}{\mu\,(2\,n-n_{\prime})}\right\}\frac{n\,e_{\prime}}{(2\,n-n_{\prime})}\sin\left(2\,n\,t-n_{\prime}\,t+2\,\varepsilon-\varepsilon_{\prime}-\varpi_{\prime}\right)$$

$$+\left\{2\,r_{18}-\frac{3\,m_{\prime}\,a\,R_{18}\,n}{\mu\,(3\,n-2\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(3\,n-2\,n_{\prime})}\sin\left(3\,n\,t-2\,n_{\prime}\,t+3\,\varepsilon-2\,\varepsilon_{\prime}-\varpi_{\prime}\right)$$

$$+\left\{2\,r_{19}-\frac{4\,a\,R_{19}\,n}{\mu\,(4\,n-3\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(4\,n-3\,n_{\prime})}\sin\left(4\,n\,t-3\,n_{\prime}\,t+4\,\varepsilon-3\,\varepsilon_{\prime}-\varpi_{\prime}\right)$$

$$+\left\{2\,r_{20}-\frac{m_{\prime}\,a\,R_{20}\,n}{\mu\,(n-2\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(n-2\,n_{\prime})}\sin\left(n\,t-2\,n_{\prime}\,t+\varepsilon-2\,\varepsilon_{\prime}+\varpi_{\prime}\right)$$

$$+\left\{2\,r_{21}-\frac{2\,m_{\prime}\,a\,R_{21}\,n}{\mu\,(2\,n-3\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(2\,n-3\,n_{\prime})}\sin\left(2\,n\,t-3\,n_{\prime}\,t+2\,\varepsilon-2\,\varepsilon_{\prime}+\varpi_{\prime}\right)$$

$$+\left\{2\,r_{22}-\frac{3\,m_{\prime}\,a\,R_{22}\,n}{\mu\,(3\,n-4\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(3\,n-4\,n_{\prime})}\sin\left(3\,n\,t-4\,n_{\prime}\,t+3\,\varepsilon-4\,\varepsilon_{\prime}+\varpi_{\prime}\right)$$

$$+\left\{2\,r_{23}-\frac{4\,m_{\prime}\,a\,R_{23}\,n}{\mu\,(4\,n-5\,n_{\prime})}\right\}\frac{n\,e_{\prime}}{(4\,n-5\,n_{\prime})}\sin\left(4\,n\,t-5\,n_{\prime}\,t-4\,\varepsilon-5\,\varepsilon_{\prime}+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{24}+\frac{r_{12}}{2}\right)\frac{m_{\prime}}{\mu}\left\{\frac{a\,R_{24}\,n}{m_{\prime}}+\frac{2\,a\,R_{12}\,n}{(n-2\,n_{\prime})}\right\}\right\}\frac{n\,e^2}{n_{\prime}}\sin\left(2\,n_{\prime}\,t+2\,\varepsilon_{\prime}-2\,\varpi\right)$$

$$+\left\{2\left(r_{25}+\frac{r_6}{2}+\frac{r_{\prime}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{a\,R_{25}\,n}{(n+n_{\prime})}-\frac{a\,R_6\,n}{n_{\prime}}+\frac{5\,a\,R_1\,n}{4\,(n-n_{\prime})}\right\}\right\}\frac{n\,e^2}{n_{\prime}}\sin\left(nt+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-2\,\varpi\right)$$

$$+\left\{2\left(r_{27}+\frac{r_8}{2}+\frac{r_{\prime}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{a\,R_{27}\,n}{(3\,n-n_{\prime})}+\frac{a\,R_8\,n}{(2\,n-n_{\prime})}+\frac{5\,a\,R_1\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(3\,n-n_{\prime})}\sin\left(3\,n\,t-n_{\prime}t+3\,\varepsilon-\varepsilon_{\prime}-2\varpi\right)$$

Expression for the longitude.

$$+\left\{2\left(r_{28}+\frac{r_9}{2}+\frac{r_2}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{2\,a\,R_{28}\,n}{(4\,n-2\,n_{\prime})}+\frac{2\,a\,R_9\,n}{(3\,n-2\,n_{\prime})}+\frac{5\,a\,R_2\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(4\,n-2\,n_{\prime})}\sin\left(4\,n\,t-2\,n_{\prime}\,t+4\,\varepsilon-2\,\varepsilon_{\prime}-2\,\varpi\right)$$

$$+\left\{2\left(r_{29}+\frac{r_{10}}{2}+\frac{r_3}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{29}\,n}{(5\,n-3\,n_{\prime})}+\frac{3\,a\,R_{10}\,n}{(4\,n-3\,n_{\prime})}+\frac{5\,a\,R_3\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(5\,n-3\,n_{\prime})}\sin\left(5\,n\,t-3\,n_{\prime}t+5\,\varepsilon-3\,\varepsilon_{\prime}-2\,\varpi\right)$$

$$+\left\{2\left(r_{30}+\frac{r_{11}}{2}+\frac{r_4}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{30}\,n}{(6\,n-4\,n_{\prime})}+\frac{4\,a\,R_{11}\,n}{(5\,n-4\,n_{\prime})}+\frac{5\,a\,R_4\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(6\,n-4\,n_{\prime})}\sin\left(6\,nt-4\,n_{\prime}t+6\,\varepsilon-4\,\varepsilon_{\prime}-2\,\varpi\right)$$

$$+\left\{2\left(r_{32}+\frac{r_{13}}{2}+\frac{r_3}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{32}\,n}{(n-3\,n_{\prime})}+\frac{3\,a\,R_{13}\,n}{(2\,n-3\,n_{\prime})}+\frac{5\,a\,R_3\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(n-3\,n_{\prime})}\sin\left(n\,t-3\,n_{\prime}t+\varepsilon-3\,\varepsilon_{\prime}+2\,\varpi\right)$$

$$+\left\{2\left(r_{33}+\frac{r_{14}}{2}+\frac{r_4}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{33}\,n}{(2\,n-4\,n_{\prime})}+\frac{4\,a\,R_{14}\,n}{(3\,n-4\,n_{\prime})}+\frac{5\,a\,R_4\,n}{4\,(n-n_{\prime})}\right\}\right\}$$

$$\frac{n\,e^2}{(2\,n-4\,n_{\prime})}\sin\left(2nt-4\,n_{\prime}t+2\varepsilon-4\,\varepsilon_{\prime}+2\,\varpi\right)$$

$$+\left\{2\left(r_{36}+\frac{r_{20}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{a\,R_{36}\,n}{(2\,n-2\,n_{\prime})}+\frac{a\,R_{20}\,n}{(n-2\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(2\,n-2\,n_{\prime})}\sin\left(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}-\varpi+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{37}+\frac{r_{21}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{2\,a\,R_{37}\,n}{(3n-3n_{\prime})}+\frac{2\,a\,R_{21}\,n}{(2\,n-3\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(3n-3n_{\prime})}\sin\left(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}-\varpi+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{38}+\frac{r_{22}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{38}\,n}{(4n-4n_{\prime})}+\frac{3\,a\,R_{22}\,n}{(3n-4n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(4n-4n_{\prime})}\sin\left(4nt-4n_{\prime}t+4\varepsilon-4\varepsilon_{\prime}-\varpi+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{39}+\frac{r_{23}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{39}\,n}{(5n-5n_{\prime})}+\frac{4\,a\,R_{23}\,n}{(4n-5\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(5\,n-5\,n_{\prime})}\sin\left(5\,nt-5\,n_{\prime}t+5\,\varepsilon-5\varepsilon_{\prime}-\varpi+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{43}+\frac{r_{18}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{43}\,n}{(2\,n-2\,n_{\prime})}+\frac{3\,a\,R_{18}\,n}{(3\,n-2\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(2\,n-2\,n_{\prime})}\sin\left(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}+\varpi-\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{44}+\frac{r_{19}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{44}\,n}{(3\,n-3\,n_{\prime})}+\frac{4\,a\,R_{19}\,n}{(4\,n-3\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(3\,n-3\,n_{\prime})}\sin\left(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}+\varpi-\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{46}+\frac{r_{20}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{-\frac{a\,R_{46}\,n}{2\,n_{\prime}}+\frac{a\,R_{20}\,n}{(n-2\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{2\,n_{\prime}}\sin\left(2\,n_{\prime}t+2\,\varepsilon-\varpi-\varpi_{\prime}\right)$$

Expression for the longitude.

$$+\left\{2\left(r_{49}+\frac{r_{17}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{2\,a\,R_{49}\,n}{(3\,n-n_{\prime})}+\frac{2\,a\,R_{17}\,n}{(2\,n-n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(3\,n-n_{\prime})}\sin\left(3\,n\,t-n_{\prime}t+3\,\varepsilon-\varepsilon_{\prime}-\varpi-\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{50}+\frac{r_{18}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{50}\,n}{(4\,n-2\,n_{\prime})}+\frac{3\,a\,R_{18}\,n}{(3\,n-2\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(4\,n-2\,n_{\prime})}\sin(4nt-2n_{\prime}t+4\varepsilon-2\varepsilon_{\prime}-\varpi-\varpi_{\prime})$$

$$+\left\{2\left(r_{51}+\frac{r_{19}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{51}\,n}{(5\,n-3\,n_{\prime})}+\frac{4\,a\,R_{19}\,n}{(4\,n-3\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(5\,n-3\,n_{\prime})}\sin\left(5\,nt-3\,n_{\prime}t+5\varepsilon-3\varepsilon_{\prime}-\varpi-\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{53}+\frac{r_{21}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{2\,a\,R_{53}\,n}{(n-3\,n_{\prime})}+\frac{2\,a\,R_{21}\,n}{(2\,n-3\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(n-3\,n_{\prime})}\sin\left(n\,t-3\,n_{\prime}t+e-3\,\varepsilon_{\prime}+\varpi+\varpi_{\prime}\right)$$

$$+\left\{2\left(r_{54}+\frac{r_{22}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{3\,a\,R_{54}\,n}{(2n-4\,n_{\prime})}+\frac{3\,a\,R_{22}\,n}{(3\,n-4\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(2\,n-4\,n_{\prime})}\sin(2nt-4n_{\prime}t+2\varepsilon-4\varepsilon_{\prime}+\varpi+\varpi_{\prime})$$

$$+\left\{2\left(r_{55}+\frac{r_{23}}{2}\right)-\frac{m_{\prime}}{\mu}\left\{\frac{4\,a\,R_{55}\,n}{(3\,n-5\,n_{\prime})}+\frac{4\,a\,R_{23}\,n}{(4\,n-5\,n_{\prime})}\right\}\right\}\frac{n\,e\,e_{\prime}}{(3\,n-5\,n_{\prime})}\sin(3nt-5n_{\prime}t+3\varepsilon-5\varepsilon_{\prime}+\varpi+\varpi_{\prime})$$

$$+\left\{2\,r_{58}-\frac{m_{\prime}}{\mu}\frac{a\,R_{58}\,n}{(n+n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(n+n_{\prime})}\sin\left(n\,t+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{59}-\frac{m_{\prime}\,a\,R_{59}}{\mu}\right\}e_{\prime}^{2}\sin\left(2\,n\,t+2\,\varepsilon-2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{60}-\frac{m_{\prime}}{\mu}\frac{3\,a\,R_{60}\,n}{(3\,n-n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(3\,n-n_{\prime})}\sin\left(3\,n\,t-n_{\prime}t+3\,\varepsilon-\varepsilon_{\prime}-2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{61}-\frac{m_{\prime}}{\mu}\frac{4\,a\,R_{61}\,n}{(4\,n-2\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(4\,n-2\,n_{\prime})}\sin\left(4\,n\,t-2\,n_{\prime}t+4\,\varepsilon-2\,\varepsilon_{\prime}-2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{62}-\frac{m_{\prime}}{\mu}\frac{5\,a\,R_{62}\,n}{(5\,n-3\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(5\,n-3\,n_{\prime})}\sin 5\,n\,t-3\,n_{\prime}t+5\,\varepsilon-3\,\varepsilon_{\prime}-2\,\varpi_{\prime})$$

$$+\left\{2\,r_{63}-\frac{m_{\prime}}{\mu}\frac{a\,R_{63}\,n}{(n-3\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(n-3\,n_{\prime})}\sin\left(n\,t-3\,n_{\prime}t+\varepsilon-3\,\varepsilon_{\prime}+2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{64}-\frac{m_{\prime}}{\mu}\frac{2\,a\,R_{64}\,n}{(2\,n-4\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(2\,n-4\,n_{\prime})}\sin\left(2\,n\,t-4\,n_{\prime}t+2\,\varepsilon-4\,\varepsilon_{\prime}+2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{65}-\frac{m_{\prime}}{\mu}\frac{3\,a\,R_{65}\,n}{(3\,n-5\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(3\,n-5\,n_{\prime})}\sin\left(3\,n\,t-5\,n_{\prime}t+3\,\varepsilon-5\,\varepsilon_{\prime}+2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{66}-\frac{m_{\prime}}{\mu}\frac{4\,a\,R_{66}\,n}{(4\,n-6\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(4\,n-6\,n_{\prime})}\sin\left(4\,n\,t-6\,n_{\prime}t+4\,\varepsilon-6\,\varepsilon_{\prime}+2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{67}-\frac{m_{\prime}}{\mu}\frac{5\,a\,R_{67}\,n}{(5\,n-7\,n_{\prime})}\right\}\frac{n\,e_{\prime}^{2}}{(5\,n-7\,n_{\prime})}\sin\left(5\,n\,t-7\,n_{\prime}t+5\,\varepsilon-7\,\varepsilon_{\prime}+2\,\varpi_{\prime}\right)$$

In order to convert the coefficients of the inequalities of the longitude into sexagesimal seconds, they must be multiplied by $\frac{1296000}{2\pi}$, the logarithm of which number is 5·3144251, the corresponding logarithm for centesimal seconds is 5·8038801.

$$\frac{\mathrm{d}R}{\mathrm{d}z} = m_{\prime}\left\{\frac{z_{\prime}}{r_{\prime}^{3}} + \frac{z - z_{\prime}}{\{r^{2} - 2rr_{\prime}\cos(\lambda - \lambda_{\prime}) + r_{\prime}^{2}\}^{\frac{3}{2}}}\right\}$$

If $z = 0$, and the products $e \tan \iota_{\prime}$, $e_{\prime} \tan \iota_{\prime}$ be neglected,

$$\frac{\mathrm{d}R}{\mathrm{d}z} = \frac{m_{\prime}}{a_{\prime}^{2}}\tan\iota_{\prime}\sin(n_{\prime}t + \varepsilon_{\prime} - \nu_{\prime})\{1 - b_{3,0} - b_{3,1}\cos(nt - nt + \varepsilon - \varepsilon_{\prime}) - b_{3,2}\cos(2nt - 2n_{\prime}t + 2\varepsilon - 2\varepsilon_{\prime})\}$$

$$\frac{a^{3}\,\mathrm{d}^{2}s}{\mathrm{d}t^{2}} + \mu s + \frac{m_{\prime}}{a_{\prime}^{2}}a^{2}\tan\iota_{\prime}\sin(n_{\prime}t + \varepsilon_{\prime} - \nu_{\prime})\{1 - b_{3,0} - b_{3,1}\cos(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime}) - \&c.\} = 0$$

Expression for the tangent of the latitude.

$$s = -\frac{m_{\prime}}{\mu}\frac{n^{2}}{(n - n_{\prime})(n + n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}\{1 - b_{3,0}\}\sin(n_{\prime}t + \varepsilon_{\prime} - \nu_{\prime})$$

$$+ i\sin\Big((1 + l)nt + \varepsilon - \nu_{\prime}\Big)$$

$$-\frac{m_{\prime}}{\mu}\frac{n^{2}}{4n_{\prime}(2n - 2n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,1}\sin(nt - 2n_{\prime}t + \varepsilon - 2\varepsilon_{\prime} + \nu_{\prime})$$

$$-\frac{m_{\prime}}{\mu}\frac{n^{2}}{2(n - n_{\prime})(3n - n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,2}\sin(2nt - n_{\prime}t + 2\varepsilon - \varepsilon_{\prime} - \nu_{\prime})$$

$$+\frac{m_{\prime}}{\mu}\frac{n^{2}}{2(n - 3n_{\prime})(3n - 3n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,2}\sin(2nt - 3n_{\prime}t + 2\varepsilon - 3\varepsilon_{\prime} + \nu_{\prime})$$

$$-\frac{m_{\prime}}{\mu}\frac{n^{2}}{2(2n - 2n_{\prime})(4n - 2n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,3}\sin(3nt - 2n_{\prime}t + 3\varepsilon - 2\varepsilon_{\prime} - \nu_{\prime})$$

$$+\frac{m_{\prime}}{\mu}\frac{n^{2}}{2(2n - 4n_{\prime})(4n - 4n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,4}\sin(3nt - 4n_{\prime}t + 3\varepsilon - 4\varepsilon_{\prime} + \nu_{\prime})$$

$$-\frac{m_{\prime}}{\mu}\frac{n^{2}}{2(3n - 3n_{\prime})(5n - 3n_{\prime})}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,4}\sin(4nt \quad 3n_{\prime}t + 4\varepsilon - 3\varepsilon_{\prime} - \nu_{\prime})$$

$$l(2 + l)\,i = \frac{m}{2\mu_{\prime}}\frac{a^{2}}{a_{\prime}^{2}}\tan\iota_{\prime}b_{3,1}$$

$$R_{6} = -\frac{3a}{2a_{\prime}^{2}} + \frac{3a}{2a_{\prime}^{2}}b_{3,0} - \frac{a^{2}}{2a_{\prime}^{3}}b_{3,1} - \frac{a}{4a_{\prime}^{2}}b_{3,2}$$ (See p. 31.)

$$q_{6} = -\frac{3a}{2a_{\prime}^{2}} + \frac{3}{2}\frac{a}{a_{\prime}^{2}}b_{3,0} - \frac{a^{2}}{2a_{\prime}^{3}}b_{3,1} - \frac{a}{4a_{\prime}^{2}}b_{3,2} - \frac{3.3a^{2}}{2a_{\prime}^{3}}\left\{\frac{a}{a_{\prime}}b_{5,0} - \tfrac{1}{2}b_{5,1}\right\}$$

$$+\frac{3}{2}\frac{a^{3}}{a_{\prime}^{4}}\left\{\frac{a}{a_{\prime}}b_{5,1} - b_{5,0} - \tfrac{1}{2}b_{5,2}\right\} + \frac{3}{4}\frac{a^{2}}{a_{\prime}^{3}}\left\{\frac{a}{a_{\prime}}b_{5,2} - \tfrac{1}{2}b_{5,1} - \tfrac{1}{2}b_{5,3}\right\}$$

$$q_6 = -\frac{3\,a}{2\,a_{,}^2} + \frac{3}{2}\,\frac{a}{a_{,}^2}\,b_{3,0} - \frac{a^2}{2\,a_{,}^3}b_{3,1} - \frac{a}{4\,a_{,}^2}b_{3,2} + \frac{3}{2}\,\frac{a^2}{a_{,}^3}\left\{\left(1 + \frac{a^2}{a_{,}^2}\right)b_{5,1} - 2\,\frac{a}{a_{,}}\,b_{5,0} - \frac{a}{a_{,}^2}b_{5,2}\right\}$$

$$-\frac{3\,a^3}{a_{,}^4}\left\{b_{5,0} - \tfrac{1}{2}\,b_{5,2}\right\} + \frac{3}{8}\,\frac{a^2}{a_{,}^3}\left\{b_{5,1} - b_{5,3}\right\}$$

$$= -\frac{3\,a}{2\,a_{,}^2} + \frac{3}{2}\,\frac{a}{a_{,}^2}\,b_{3,0} - \frac{a^2}{2\,a_{,}^3}b_{3,1} - \frac{a}{4\,a_{,}^2}\,b_{3,2} + \frac{3}{2}\,\frac{a^2}{a_{,3}}\,b_{3,1} - \frac{a^2}{a_{,3}}\,b_{3,1} + \frac{2}{4}\,\frac{a}{a_{,}^2}\,b_{3,2}$$

$$= -\frac{3\,a}{2\,a_{,}^2} + \frac{3}{2}\,\frac{a}{a_{,}^2}\,b_{3,0} - \frac{2\,a^2}{2\,a_{,}^3}\,b_{3,1} - \frac{a}{4\,a_{,}^2}\,b_{3,2}$$

The quantities of which the general symbol is q, and which refer to the terms in the development of R multiplied by the eccentricities, admit of similar reductions; so that

$$q_7 = -\frac{a^2}{a_{,}^3}b_{3,0} + \frac{a}{a_{,}^2}\,b_{3,1} \qquad q_{16} = -\frac{a}{2\,a_{,}^2}\,b_{3,2}$$

Considering only the terms in $2\int \mathrm{d}R + r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)$, of which the arguments are $n\,t + \varepsilon - \varpi$, and $n\,t + \varepsilon - \varpi_{,}$

$$2\int \mathrm{d}\,R + r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right) = m_{,}\,q_7\,e\cos\left(n\,t + \varepsilon - \varpi\right) + m_{,}\,q_{16}\,e_{,}\cos\left(n\,t + \varepsilon - \varpi_{,}\right)$$

$$= m_{,}\,\mathrm{q}\cos\left(n\,t + \varepsilon - \varpi_1\right)$$

provided

$$\mathrm{q}\cos\varpi_1 = q_7\,e\cos\varpi + q_{16}\,e_{,}\cos\varpi_{,}$$

$$\mathrm{q}\cos\varpi_1 = q_7\,e\sin\varpi + q_{16}\,e_{,}\sin\varpi_{,}$$

And if

$$\frac{a}{r} = 1 + r_0 + e\cos\left(n\left(1 + k\right)t + \varepsilon - \varpi_1\right) + \text{\&c.}$$

$$(1 + k)^2\,(1 - 3\,r_0) - 1 + \frac{m_{,}\,a}{\mu\,e}\,\mathrm{q} = 0 \qquad k = \frac{3}{2}\,r_0 - \frac{m_{,}\,a}{2\,\mu\,e}\,\mathrm{q}$$

$$r_0 = \frac{m_{,}}{\mu}\left\{\frac{a^3}{a_{,}^3}\,b_{3,0} - \frac{a^2}{2\,a_{,}^2}\,b_{3,1}\right\}$$

$$\mathrm{q}^2 = q_7^{\,2}\,e^2 + 2\,q_7\,q_{16}\,e\,e_{,}\cos\left(\varpi - \varpi_{,}\right) + q_{16}^{\,2}\,e_{,}^{\,2}$$

$$\mathrm{q} = q_7\,e + q_{16}\,e_{,}\cos\left(\varpi - \varpi_{,}\right) \text{ nearly}$$

$$\frac{\mathrm{q}}{e} = -\frac{a^2}{a_{,}^3}\,b_{3,0} + \frac{a}{a_{,}^2}\,b_{3,1} - \frac{a}{2\,a_{,}^2}\,\frac{e_{,}}{e}\,b_{3,2}\cos\left(\varpi - \varpi_{,}\right)$$

$$k = \frac{m_{,}}{\mu}\left\{\frac{3\,a^3}{2\,a_{,}^3}\,b_{3,0} - \frac{3\,a^2}{4\,a_{,}^2}\,b_{3,1} + \frac{a^3}{2\,a_{,}^3}\,b_{3,0} - \frac{a^2}{2\,a_{,}^2}\,b_{3,1} + \frac{a^2}{4\,a_{,}^2}b_{3,2}\,\frac{e_{,}}{e}\cos\left(\varpi - \varpi_{,}\right)\right\}$$

$$= \frac{m_{,}}{\mu}\left\{\frac{2\,a^3}{a_{,}^3}\,b_{3,0} - \frac{5}{4}\,\frac{a^2}{a_{,}^2}\,b_{3,1} + \frac{a^2}{4\,a_{,}^2}\,b_{3,2}\,\frac{e_{,}}{e}\cos\left(\varpi - \varpi_{,}\right)\right\}$$

$$\frac{d\lambda}{dt} = n(1 + 2r_0)t + \varepsilon + 2(1 + r_0)e\cos\left(n(1 + k)t + \varepsilon - \varpi_1\right)$$

$$- \frac{m_{,}}{\mu} a R_{16} e_{,} \cos(nt + \varepsilon - \varpi_{,})$$

and neglecting the square of the disturbing force

$$\frac{d\lambda}{dt} = n(1 + 2r_0)t + \varepsilon + 2(1 + r_0)e\cos\left(n(1 + k)t + \varepsilon - \varpi_{,}\right)$$

$$- \frac{m_{,}}{\mu} a R_{16} e_{,} \cos\left(n(1 + k)t + \varepsilon - \varpi_{,}\right)$$

If

$$\mathrm{e}(1 + k)\cos\varpi_2 = (1 + r_0)e\cos\varpi_1 - \frac{m_{,}}{2\mu} a R_{16} e_{,} \cos\varpi_{,}$$

$$\mathrm{e}(1 + k)\sin\varpi_2 = (1 + r_0)e\sin\varpi_1 - \frac{m_{,}}{2\mu} a R_{16} e_{,} \sin\varpi_{,}$$

$$\lambda = n(1 + 2r_0)t + \varepsilon\, 2\,\mathrm{e}\sin\left(n(1 + k)t + \varepsilon - \varpi_2\right)$$

$$e(1 + r_0)\cos\varpi_1 = \mathrm{e}(1 + k)\cos\varpi_2 + \frac{m_{,}}{2\mu} a R_{16} e_{,} \cos\varpi_{,}$$

$$e(1 + r_0)\sin\varpi_1 = \mathrm{e}(1 + k)\sin\varpi_2 + \frac{m_{,}}{2\mu} a R_{16} e_{,} \sin\varpi_{,}$$

$$e(1 + r_0) = \mathrm{e}(1 + k)\left\{1 + \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e} \cos(\varpi_2 - \varpi_{,})\right.$$

$$\cos\varpi_1 = \cos\varpi_2\left\{1 - \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\cos(\varpi_2 - \varpi_{,})\right\} + \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\cos\varpi_{,}$$

$$\sin\varpi_1 = \sin\varpi_2\left\{1 - \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\cos(\varpi_2 - \varpi_{,})\right\} + \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\sin\varpi_{,}$$

$$\sin(\varpi_2 - \varpi_1) = \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\cos\varpi_{,}$$

therefore neglecting the square of the disturbing force

$$\frac{e}{a} = \frac{\mathrm{e}}{a}\left\{1 + k - r_0 + \frac{m_{,}}{2\mu} a R_{16} \frac{e_{,}}{e}\cos(\varpi - \varpi_{,})\right\}$$

$$= \frac{\mathrm{e}}{a}\left\{1 + \frac{m_{,}}{\mu}\left\{\frac{2a^3}{a_{,}^3} b_{3,0} - \frac{5}{4}\frac{a^2}{a_{,}^2} b_{3,1} - \frac{a^3}{a_{,}^3} b_{3,0} + \frac{a^2}{2a_{,}^2} b_{3,1}\right\}\right.$$

$$\left. + \frac{m_{,}}{\mu}\left\{\frac{a^2}{4a_{,}^2} b_{3,2} + \frac{3a^2}{4a_{,}^2} b_{3,0} - \frac{a}{4a_{,}} b_{3,1} - \frac{a^2}{8a_{,}^2} b_{3,2}\right\}\frac{e_{,}}{e}\cos(\varpi - \varpi_{,})\right\}$$

$$= \frac{\mathrm{e}}{a}\left\{1 + \frac{m_{,}}{\mu}\left\{\frac{a^3}{a_{,}^3} b_{3,0} - \frac{3}{4}\frac{a^2}{a_{,}^2} b_{3,1} - \frac{a^3}{4a_{,}^3} b_{3,1}\frac{e_{,}}{e}\cos(\varpi - \varpi_{,})\right\}\right\}$$

Let

$$n\left\{1+\frac{2m_{\prime}}{\mu}\left(\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{a^2}{2a_{\prime}^2}b_{3,1}\right)\right\}=\mathrm{n}$$

$$n(1+k)=\mathrm{n}\left\{1-\frac{m\ a^2}{4\mu_{\prime}a_{\prime}^2}b_{3,1}+\frac{m_{\prime}a^2e_{\prime}}{4\mu\,a_{\prime}^2e}b_{3,2}\cos(\varpi-\varpi_{\prime})\right\}$$

Let $\frac{\mu}{\mathrm{a}^3}=\mathrm{n}^2$, then $a^3=\mathrm{a}^3\left\{1+\frac{4m_{\prime}}{\mu}\left(\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{a^2}{a_{\prime}^2}b_{3,1}\right)\right\}$

$$e=\mathrm{a}\left\{1+\frac{4m_{\prime}}{3\mu}\left(\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{a^2}{a_{\prime}^2}b_{3,1}\right)\right\}$$

$$\lambda=\mathrm{n}t+\varepsilon+2\,\mathrm{e}\sin\left(\mathrm{n}\left\{1-\frac{m_{\prime}a^2}{4\mu\,a_{\prime}^2}b_{3,1}+\frac{m_{\prime}}{4\mu_{\prime}}\frac{a^2}{a_{\prime}^2}\frac{e_{\prime}}{e}b_{3,2}\cos(\varpi-\varpi_{\prime})\right\}t+\varepsilon-\varpi_2\right)+\&c.$$

$$\frac{\mathrm{a}}{r}=1-\frac{m_{\prime}}{3\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{a^2}{2a_{\prime}^2}b_{3,1}\right\}$$

$$+\mathrm{e}\left\{1-\frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0}+\frac{a^2}{12a_{\prime}^2}b_{3,1}-\frac{a^2}{4a_{\prime}^2}b_{3,2}\frac{e_{\prime}}{e}\cos(\varpi-\varpi_{\prime})\right\}\right\}$$

$$\cos\left(\mathrm{n}\left\{1-\frac{m_{\prime}}{4\mu}\frac{a^2}{a_{\prime}^2}b_{3,1}+\frac{m_{\prime}}{4\mu}\frac{a^2}{a_{\prime}^2}\frac{e_{\prime}}{e}b_{3,2}\cos(\varpi-\varpi_{\prime})\right\}t+\varepsilon-\varpi_2\right)$$

$$+\frac{m_{\prime}}{\mu}\left\{\frac{3}{4}\frac{a^2}{a_{\prime}^2}b_{3,0}-\frac{a}{4a_{\prime}}b_{3,1}-\frac{a^2}{8a_{\prime}^2}b_{3,2}\right\}e_{\prime}\cos(nt+\varepsilon-\varpi_{\prime})$$

$$\frac{r}{\mathrm{a}}=1+\frac{m_{\prime}}{3\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{a^2}{2a_{\prime}^2}b_{3,1}\right\}$$

$$-e\left\{1+\frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0}-\frac{5}{12}\frac{a^2}{a_{\prime}^2}b_{3,1}+\frac{a^2}{a_{\prime}^2}b_{3,2}\frac{e_{\prime}}{e}\cos(\varpi-\varpi_{\prime})\right\}\right\}$$

$$\cos\left(\mathrm{n}\left\{1-\frac{m_{\prime}}{4\mu}\frac{a^2}{a_{\prime}^2}b_{3,1}+\frac{m_{\prime}}{4\mu}\frac{a^2}{a_{\prime}^2}\frac{e_{\prime}}{e}b_{3,2}\cos(\varpi-\varpi_{\prime})\right\}t+\varepsilon-\varpi_2\right)$$

$$-\frac{m_{\prime}}{\mu}\left\{\frac{3}{4}\frac{a^2}{a_{\prime}^2}b_{3,0}-\frac{a}{4a_{\prime}}b_{3,1}-\frac{a^2}{8a_{\prime}^2}b_{3,2}\right\}e_{\prime}\cos(nt+\varepsilon-\varpi_{\prime})+\&c.$$

If

$$\frac{a}{r}=1+r_0+e(1+f)\cos\left(n(1+k^*)t+\varepsilon-\varpi\right)+e_{\prime}f_{\prime}\cos\left(n(1+k_{\prime})t+\varepsilon-\varpi_{\prime}\right)+\&c.$$

$$(1+f)\left\{(1+k)^2(1-3r_0)-1\right\}+\frac{m_{\prime}}{\mu}a\,q_7=0$$

* This quantity k must not be confounded with the quantity k above.

whence neglecting k^2, kf, &c.

$$k = \frac{3}{2} r_0 - \frac{m_{\prime}}{2\mu} a q_7$$

$$= \frac{m_{\prime}}{\mu} \left\{ \frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{a^2}{2 a_{\prime}^2} b_{3,1} \right\}$$

similarly

$$f_{\prime} \{(1 + k_{\prime})^2 - 1\} + \frac{m_{\prime}}{\mu} a q_{16} = 0$$

$$k_{\prime}^2 + 2 k_{\prime} + \frac{m_{\prime}}{\mu} \frac{a}{f_{\prime}} q_{16} = 0, \qquad q_{16} = -\frac{a}{2 a_{\prime}^2} b_{3,2}$$

$$k_{\prime} = -2 - \frac{m_{\prime}}{4\mu} \frac{a^2}{f_{\prime} a_{\prime}^2} b_{3,2} \qquad \text{or} + \frac{m_{\prime}}{4\mu} \frac{a^2}{a_{\prime}^2 f_{\prime}} b_{3,2}$$

$$\lambda = n\{1 + 2 r_0\} t + \varepsilon + \frac{2 e (1 + f)(1 + r_0)}{(1 + k)} \sin \Big(n (1 + k) t + \varepsilon - \varpi \Big)$$

$$+ 2 e_{\prime} \left\{ \frac{f_{\prime}}{1 + k_{\prime}} - \frac{m_{\prime} a}{2\mu} R_{16} \right\} \sin \Big(n (1 + k_{\prime}) t + \varepsilon - \varpi_{\prime} \Big) + \text{\&c. nearly}$$

If

$$\frac{e(1 + f)(1 + r_0)}{1 + k} = \mathrm{e}$$

$$(1 + f) = \mathrm{e}\,(1 + k - r_0)$$

If $\frac{f_{\prime}}{1 + k_{\prime}} - \frac{m_{\prime}}{2\mu} a R_{16} = 0$ and if $k_{\prime}^2$ be neglected

$$f_{\prime} = \frac{m_{\prime}}{4\mu} \frac{a^2}{a_{\prime}^2} b_{3,2} + \frac{m_{\prime}}{2\mu} a R_{16}$$

If $n (1 + 2 r_0) = \mathrm{n}$, and $\mathrm{n}^2 = \frac{\mu}{\mathrm{a}^3}$

$$a = \mathrm{a} \left\{ 1 + \frac{4}{3} r_0 \right\}$$

$$\frac{\mathrm{a}}{r} = 1 - \frac{1}{3} r_0 + \mathrm{e} \left\{ 1 + k - \frac{7}{3} r_0 \right\} \cos \Big(n (1 + k) t + \varepsilon - \varpi \Big) + \mathrm{e}_{\prime} f_{\prime} \cos \Big(n (1 + k_{\prime}) t + \varepsilon - \varpi_{\prime} \Big)$$

$$= 1 - \frac{1}{3} r_0 + \mathrm{e} \left\{ 1 - \frac{5}{6} r_0 - \frac{m_{\prime}}{2\mu} a q_7 \right\} \cos \Big(\mathrm{n} (1 + \frac{1}{2} r_0 - \frac{m_{\prime}}{2\mu} a q_7) t + \varepsilon - \varpi \Big)$$

$$- e_{\prime} \left\{ \frac{m_{\prime}}{2\mu} a q_{16} - \frac{m_{\prime}}{2\mu} a R_{16} \right\} \cos \Big(n (1 + k_{\prime}) t + \varepsilon - \varpi_{\prime} \Big)$$

$$\frac{r}{\mathrm{a}} = 1 + \frac{1}{3} r_0 - \mathrm{e} \left\{ 1 - \frac{1}{6} r_0 - \frac{m_{\prime}}{2\mu} a q_7 \right\} \cos \Big(\mathrm{n} (1 - \frac{1}{2} r_0 - \frac{m_{\prime}}{2\mu} a q_7) t + \varepsilon - \varpi \Big)$$

$$+ \mathrm{e} \left\{ \frac{m_{\prime}}{2\mu} a q_{16} - \frac{m_{\prime}}{2\mu} a R_{16} \right\} \cos \Big(n (1 + k_{\prime}) t + \varepsilon - \varpi_{\prime} \Big)$$

$$\frac{r}{a} = 1 + \frac{1}{3} r_0 - e\left\{1 - \frac{1}{6} r_0 - \frac{m_{\prime}}{2\mu} a q_7\right\} \cos(n t + \varepsilon - \varpi) + e_{\prime}\left\{\frac{m_{\prime}}{2\mu} a q_{16} - \frac{m_{\prime}}{2\mu} a R_{16}\right\} \cos(n t + \varepsilon - \varpi_{\prime})$$

$$- e\left\{\frac{1}{2} r_0 + \frac{m_{\prime}}{2\mu} a q_7\right\} n t \sin(n t + \varepsilon - \varpi) - e_{\prime} \frac{m_{\prime}}{2\mu} a q_{16} n t \sin(n t + \varepsilon - \varpi_{\prime})$$

In the notation of the Mécanique Céleste

$$r_0 = \frac{a^2}{2}\left(\frac{d A^{(0)}}{d a}\right) \qquad a q_7 = -\frac{a^3}{2}\left(\frac{d^2 A^{(0)}}{d a^2}\right) - \frac{3 a^2}{2}\left(\frac{d A^{(0)}}{d a}\right)$$

$$a q_{16} = -\frac{1}{2}\left\{a^2 a'\left(\frac{d^2 A^{(1)}}{d a\, d a'}\right) + 2 a^2\left(\frac{d A^{(1)}}{d a}\right) + 2 a' a\left(\frac{d A^{(1)}}{d a'}\right) + 4 a A^{(1)}\right\}$$

$$= -\frac{1}{2}\left\{2 a A - 2 a^2\left(\frac{d A^{(1)}}{d a}\right) - a^3\left(\frac{d^2 A^{(1)}}{d a^2}\right)\right\}$$

$$R_{16} = -\frac{1}{2} a'\left(\frac{d A^{(1)}}{d a'}\right) - A^{(1)}$$

$$= \frac{a}{2}\left(\frac{d A^{(1)}}{d a}\right) - \frac{1}{2} A^{(1)}$$

$$\frac{\mu}{6 m_{\prime}} r_0 + \frac{a}{2} q_7 = \frac{a^2}{12}\left(\frac{d A^{(0)}}{d a}\right) - \frac{a^3}{4}\left(\frac{d^2 A^{(0)}}{d a^2}\right) - \frac{3}{4} a^2\left(\frac{d A^{(0)}}{d a}\right)$$

$$= -\frac{2 a^2}{3}\left(\frac{d A^{(0)}}{d a}\right) - \frac{a^3}{4}\left(\frac{d^2 A^{(0)}}{d a^2}\right) = -f$$

$$\frac{a}{2} q_{16} - \frac{a}{2} R_{16} = -\frac{a A^{(1)}}{2} + \frac{a^2}{2}\left(\frac{d A^{(1)}}{d a}\right) + \frac{a^3}{4}\left(\frac{d^2 A^{(1)}}{d a^2}\right) + \frac{a A^{(1)}}{4} - \frac{a^2}{4}\left(\frac{d A^{(1)}}{d a}\right)$$

$$= -\frac{a A^{(1)}}{4} + \frac{a^2}{4}\left(\frac{d A^{(1)}}{d a}\right) \overset{*}{+} \frac{a^3}{4}\left(\frac{d^2 A^{(1)}}{d a^2}\right) = -f'$$

$$\frac{\mu}{2 m_{\prime}} r_0 + a q_7 = \frac{a^2}{4}\left(\frac{d A^{(0)}}{d a}\right) - \frac{a^3}{4}\left(\frac{d^2 A^{(0)}}{d a^2}\right) - \frac{3}{4} a^2\left(\frac{d A^{(0)}}{d a}\right)$$

$$= -\frac{a^2}{2}\left(\frac{d A^{(0)}}{d a}\right) - \frac{a^3}{4}\left(\frac{d^2 A^{(0)}}{d a^2}\right) = -\frac{C}{2} = \frac{a^2}{4 a_{\prime}^2} b_{3,1}$$

$$-\frac{1}{2} a q_{16} = \frac{1}{2}\left\{a A - a^2\left(\frac{d A^{(1)}}{d a}\right) - \frac{a^3}{2}\left(\frac{d^2 A^{(1)}}{d a^2}\right)\right\} = -\frac{D}{2} = \frac{a^2}{2 a_{\prime}^2} b_{3,2}$$

These results evidently agree with those given in the Mécanique Céleste, vol. i. p. 279, with the exception of the sign in the value of f' marked with an asterisk, which I think requires alteration in that work.

Finally, neglecting the quantities multiplied by t, which may be made to depend upon the secular inequalities of the constants ε, ϖ, &c., and the squares of the eccentricities.

$$\frac{r}{a} = 1 + \frac{m_{\prime}}{3\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0} - \frac{a^2}{2a_{\prime}^2}b_{3,1}\right\} - e\left\{1 + \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3}b_{3,0} - \frac{5}{12}\frac{a^2}{a_{\prime}^2}b_{3,1}\right\}\right\}\cos(nt + \varepsilon - \varpi)$$

$$- \left\{\frac{3}{4}\frac{a^2}{a_{\prime}^2}b_{3,0} - \frac{a}{4a_{\prime}}b_{3,1} + \frac{a^2}{8a_{\prime}^2}b_{3,2}\right\}e_{\prime}\cos(nt + \varepsilon - \varpi_{\prime})$$

$$- r_1\cos(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime}) - r_2\cos(2nt - 2n_{\prime}t + 2\varepsilon - 2\varepsilon_{\prime}) - r_3\cos(3nt - 3n_{\prime}t + 3\varepsilon - 3\varepsilon_{\prime}) - \&c.$$

$$- \{r_6 - r_1\}e\cos(n_{\prime}t + \varepsilon_{\prime} - \varpi) - \{r_8 - r_1\}e\cos(2nt - n_{\prime}t + 2\varepsilon - \varepsilon_{\prime} - \varpi)$$

$$- \{r_9 - r_2\}e\cos(3nt - 2n_{\prime}t + 3\varepsilon - 2\varepsilon_{\prime} - \varpi)$$

$$- \{r_{10} - r_3\}e\cos(4nt - 3n_{\prime}t + 4\varepsilon - 3\varepsilon_{\prime} - \varpi) - \{r_{11} - r_4\}e\cos(5nt - 4n_{\prime}t + 5\varepsilon - 4\varepsilon_{\prime} - \varpi)$$

$$- \{r_{12} - r_2\}e\cos(nt - 2n_{\prime}t + \varepsilon - 2\varepsilon_{\prime} + \varpi) - \{r_{13} - r_3\}e\cos(2nt - 3n_{\prime}t + 2\varepsilon - 3\varepsilon_{\prime} + \varpi)$$

$$- \{r_{14} - r_4\}e\cos(3nt - 4n_{\prime}t + 3\varepsilon - 4\varepsilon_{\prime} + \varpi) - r_{16}e_{\prime}\cos(n_{\prime}t + e_{\prime} - \varpi_{\prime}) - r_{17}e_{\prime}\cos(nt + \varepsilon - \varpi_{\prime}) - \&c.$$

The constant part of R

$$= m_{\prime}\left\{-\frac{b_{1,0}}{a_{\prime}} + \frac{a}{2a_{\prime}^2}\left(\sin^2\frac{\iota_{\prime}}{2} - \frac{e^2 + e_{\prime}^2}{4}\right)b_{3,1} + \frac{a^2}{4a_{\prime}^2}b_{3,2}\cos(\varpi - \varpi_{\prime})\right\}$$ See p. 29.

If this quantity $= -F$ according to the notation of the Théor. Anal. vol. i. p. 336.

$$d\varepsilon = (1 - \sqrt{1 - e^2})\,d\varpi - \frac{2a^2n}{\mu}\left(\frac{dF}{da}\right)dt$$

$$d\varpi = an\frac{\sqrt{1 - e^2}}{\mu e}\left(\frac{dF}{de}\right)dt$$

$$\frac{d\varepsilon - d\varpi}{dt} = \frac{m_{\prime}}{\mu}\left\{\frac{2a^2}{a_{\prime}^3}b_{3,0} - \frac{5}{4}\frac{a^2}{a_{\prime}^2}b_{3,1} + \frac{a^2}{4a_{\prime}^2}b_{3,2}\frac{e_{\prime}}{e}\cos(\varpi \quad \varpi_{\prime})\right\}$$

Let, as hitherto, (Phil. Trans. for 1830. p. 336.)

$$r = \frac{h^2\sqrt{1 + s^2}}{\mu\cos\iota^2\{\sqrt{1 + s^2} + e\cos(\lambda' - \varpi)\}} = a\{1 - e'\cos(v - a)\}$$

Fig. 1.

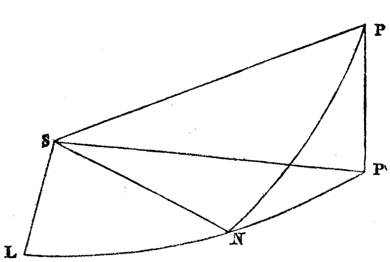
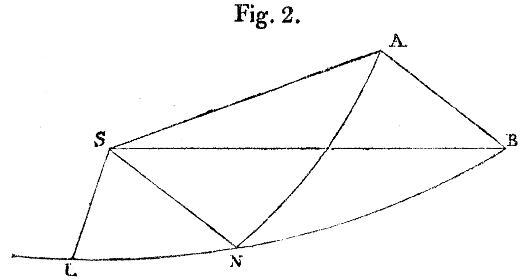


Fig. 2.



Let P be the place of the planet, P' its projection on the fixed plane L N P' (fig. 1 & 2.) S N the line of nodes, S L the line from which longitudes are reckoned. The angle L S P' $= \lambda'$. Let S A be the line of apsides. (fig. 2.)

In the notation of M. de PONTÉCOULANT, vol. i. p. 316, the angle A S N $= g$, $\frac{\varepsilon - \varpi}{n} = l$, M. de PONTÉCOULANT has given expressions for the variations of the constants a, g, e', l, ι and ν in terms of the partial differences of the quantity R with regard to these quantities. It is easy from these to find similar expressions for the variations or differentials with regard to the time of the constants a, ϖ, e, ε, ι and ν.

Let S A B be a plane cutting the plane of the orbit at right angles, so that the angle S A B $= 90^\circ$, A N B $= \iota$, B S N $= \varpi - \nu$

$$\frac{\mathrm{d}\, r^2}{\mathrm{d}\, t^2} + \frac{h^2}{r^2 \cos \iota^2} - \frac{2\,\mu}{r} + \frac{\mu}{a} = 0$$

$$r = a\,\{1 - e' \cos (v - \alpha)\}$$

When r is a maximum or minimum $\frac{\mathrm{d}\, r}{\mathrm{d}\, t} = 0$,

$$\frac{a\, h^2}{\mu \cos \iota^2} - 2\, a\, r + r^2 = 0, \quad \text{whence } r = a \pm \sqrt{a - \frac{h^2}{\mu \cos \iota^2}}$$

$$r = a\,(1 \pm e')$$

$$\frac{h^2}{\mu \cos \iota^2} = a\,(1 - e'^2)$$

By the equation of p. 336, line 12, (Phil. Trans. 1830.)

$$\frac{h^2}{\mu \cos \iota^2} = a\left(1 - e^2 + e^2 \sin^2 \iota \sin^2 (\nu - \varpi)\right)$$

$$e'^2 = e^2\,\{1 - \sin^2 \iota \sin^2 (\nu - \varpi)\}\,{}^* = e^2 \cos^2 \text{A S B}$$

Considering R first as a function of the quantities a, g, e', l, ι and ν, and then of the quantities a, ϖ, e, ε, ι and ν, we have

$$\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, a}\right) \mathrm{d}\, a + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, g}\right) \mathrm{d}\, g + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, e'}\right) \mathrm{d}\, e' + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, l}\right) \mathrm{d}\, l + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \iota}\right) \mathrm{d}\, \iota + \frac{\mathrm{d}\, R}{\mathrm{d}\, \nu} \mathrm{d}\, \nu$$

$$= \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, a}\right) \mathrm{d}\, a + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \varpi}\right) \mathrm{d}\, \varpi + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, e}\right) \mathrm{d}\, e + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \varepsilon}\right) \mathrm{d}\, \varepsilon + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \iota}\right) \mathrm{d}\, \iota + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \nu}\right) \mathrm{d}\, \nu$$

* The equation I gave, Phil. Trans. for 1830, p. 336, line 17, is not correct.

By means of this equation, the equations

$$\varepsilon - \varpi = n\,l, \quad e'^2 = e^2\{1 - \sin^2\iota \sin^2(\nu - \varpi)\}$$

and the equations given by M. DE PONTÉCOULANT, vol. i. p. 328, the values of $\frac{d\,a}{d\,t}$, $\frac{d\,\varpi}{d\,t}$, $\frac{d\,e}{d\,t}$, $\frac{d\,\varepsilon}{d\,t}$, $\frac{d\,\iota}{d\,t}$ and $\frac{d\,\nu}{d\,t}$ may be easily obtained in terms of the quantities a, ϖ, e, ε, ι, ν, and the partial differential coefficients $\left(\frac{d\,R}{d\,a}\right)$, $\left(\frac{d\,R}{d\,\varpi}\right)$, $\left(\frac{d\,R}{d\,e}\right)$, $\left(\frac{d\,R}{d\,\varepsilon}\right)$, $\left(\frac{d\,R}{d\,\iota}\right)$ and $\left(\frac{d\,R}{d\,\nu}\right)$.

Substituting in the equations of p. 40, for q_n their values and neglecting e^2, $e_{\prime}^2$, $e\,e_{\prime}$, and $\sin^2\frac{\iota_{\prime}}{2}$, when a is less than $a_{\prime}$;

Expression for the reciprocal of the radius vector, when $a < a_{\prime}$.

$$\frac{\mathrm{a}}{r} = 1 - \frac{m_{\prime}}{\mu}\left\{\frac{a^{3*}}{a_{\prime}^3}\,b_{3,0} - \frac{a^2}{a_{\prime}^2}\,b_{3,1}\right\} + \mathrm{e}\left\{1 - \frac{m_{\prime}}{\mu}\left(\frac{a^3}{a_{\prime}^3}\,b_{3,0} + \frac{a^2}{12\,a_{\prime}^2}\,b_{3,1}\right)\right\}\cos(n\,t + \varepsilon - \varpi)$$

$$+ \frac{m_{\prime}}{\mu}\,\frac{n^2}{(2n - n_{\prime})\,n_{\prime}}\left\{\frac{2n}{(n - n_{\prime})}\left(\frac{a^2}{a_{\prime}^2} - \frac{a}{a_{\prime}}\,b_{1,1}\right) + \frac{a^2}{a_{\prime}^2}\right.$$

$$\left. + \frac{a^2}{a_{\prime}^2}\left(\frac{a}{a_{\prime}}\,b_{3,1} - b_{3,0} - \tfrac{1}{2}\,b_{3,2}\right)\right\}\cos(n\,t - n_{\prime}\,t + \varepsilon - \varepsilon_{\prime}) \qquad [1]$$

$$+ \frac{m_{\prime}}{\mu}\,\frac{n^2}{(3n - 2n_{\prime})\,(n - 2n_{\prime})}\left\{\frac{2n}{(n - n_{\prime})}\,\frac{a}{a_{\prime}}\,b_{1,2}\right.$$

$$\left. - \frac{a^2}{a_{\prime}^2}\left(\frac{a}{a_{\prime}}\,b_{3,2} - \tfrac{1}{2}\,b_{3,1} - \tfrac{1}{2}\,b_{3,3}\right)\right\}\cos(2\,n\,t - 2\,n_{\prime}\,t + 2\,\varepsilon - 2\,\varepsilon_{\prime}) \qquad [2]$$

$$+ \frac{m_{\prime}}{\mu}\,\frac{n^2}{(4n - 3n_{\prime})\,(2n - 3n_{\prime})}\left\{\frac{2n}{(n - n_{\prime})}\,\frac{a}{a_{\prime}}\,b_{1,3}\right.$$

$$\left. - \frac{a^2}{a_{\prime}^2}\left(\frac{a}{a_{\prime}}\,b_{3,3} - \tfrac{1}{2}\,b_{3,2} - b_{3,4}\right)\right\}\cos(3\,n\,t - 3\,n_{\prime}\,t + 3\,\varepsilon - 3\,\varepsilon_{\prime}) \qquad [3]$$

$$+ \frac{m_{\prime}}{\mu}\,\frac{n^2}{(5n - 4n_{\prime})\,(3n - 4n_{\prime})}\left\{\frac{2n}{(n - n_{\prime})}\,\frac{a}{a_{\prime}}\,b_{1,4}\right.$$

$$\left. - \frac{a^2}{a_{\prime}^2}\left(\frac{a}{a_{\prime}}\,b_{3,4} - \tfrac{1}{2}\,b_{3,3} - \tfrac{1}{2}\,b_{3,5}\right)\right\}\cos(4\,n\,t - 4\,n_{\prime}\,t + 4\,\varepsilon + 4\,\varepsilon_{\prime}) \qquad [4]$$

$$+ \frac{m_{\prime}}{\mu}\,\frac{n^2}{(6n - 5n_{\prime})\,(4n - 5n_{\prime})}\left\{\frac{2n}{(n - n_{\prime})}\,\frac{a}{a_{\prime}}\,b_{1,5}\right.$$

$$\left. - \frac{a^2}{a_{\prime}^2}\left(\frac{a}{a_{\prime}}\,b_{3,5} - \tfrac{1}{2}\,b_{3,4} - \tfrac{1}{2}\,b_{3,6}\right)\right\}\cos(5\,n\,t - 5\,n_{\prime}\,t + 5\,\varepsilon - 5\,\varepsilon_{\prime}) \qquad [5]$$

* In the terms multiplied by $\frac{m_{\prime}}{\mu}$ the quantities a & a, e & e may be used indifferently.

Expression for the reciprocal of the radius vector, when $a < a_{\prime}$.

$$-\frac{n^2}{(n-n_{\prime})(n+n_{\prime})}\Big\{\frac{3n_{\prime}^2}{2n^2}r_1+\frac{m_{\prime}}{\mu}\Big\{\frac{3a^2}{2a_{\prime}^2}-\frac{3a^2}{2a_{\prime}^2}b_{3,0}+\frac{a^3}{2a_{\prime}^3}b_{3,1}-\frac{a^2}{4a_{\prime}^2}b_{3,2}\Big\}\Big\}e\cos(n_{\prime}t+\varepsilon_{\prime}-\varpi) \qquad [6]$$

$$\begin{aligned}&+\frac{n^2}{(3n-n_{\prime})(n-n_{\prime})}\Big\{\frac{3(2n-n_{\prime})^2}{2n^2}r_1-\frac{m_{\prime}}{\mu}\Big\{\frac{4n}{(2n-n_{\prime})}\Big\{\frac{a^2}{2a_{\prime}^2}-\frac{a^2}{2a_{\prime}^2}b_{3,0}-\frac{a^3}{2a_{\prime}^3}b_{3,1}+\frac{3a^2}{4a_{\prime}^2}b_{3,2}\Big\}\\&\qquad+\frac{a^2}{2a_{\prime}^2}-\frac{a^2}{2a_{\prime}^2}b_{3,0}+\frac{3a^3}{2a_{\prime}^3}b_{3,1}-\frac{3a^2}{4a_{\prime}^2}b_{3,2}\Big\}\Big\}e\cos(2nt-n_{\prime}t+2\varepsilon-\varepsilon_{\prime}-\varpi) \qquad [8]\end{aligned}$$

$$\begin{aligned}&+\frac{n^2}{(4n-2n_{\prime})(2n-2n_{\prime})}\Big\{\frac{3(3n-2n_{\prime})^2}{2n^2}r_2-\frac{m_{\prime}}{\mu}\Big\{\frac{6n}{(3n-2n_{\prime})}\Big\{-\frac{a^2}{4a_{\prime}^2}b_{3,1}-\frac{a^3}{2a_{\prime}^3}b_{3,2}+\frac{3a^2}{4a_{\prime}^2}b_{3,3}\Big\}\\&\qquad-\frac{a^2}{2a_{\prime}^2}b_{3,1}+\frac{5}{2}\frac{a^3}{a_{\prime}^3}b_{3,2}-\frac{3}{2}\frac{a^2}{a_{\prime}^2}b_{3,3}\Big\}\Big\}e\cos(3nt-2n_{\prime}t+3\varepsilon-2\varepsilon_{\prime}-\varpi) \qquad [9]\end{aligned}$$

$$\begin{aligned}&+\frac{n^2}{(5n-3n_{\prime})(3n-3n_{\prime})}\Big\{\frac{3(4n-3n_{\prime})^2}{2n^2}r_3-\frac{m_{\prime}}{\mu}\Big\{\frac{8n}{(4n-3n_{\prime})}\Big\{-\frac{a^2}{4a_{\prime}^2}b_{3,2}-\frac{a^3}{2a_{\prime}^3}b_{3,3}+\frac{3a^2}{4a_{\prime}^2}b_{3,4}\Big\}\\&\qquad-\frac{3a^2}{4a_{\prime}^2}b_{3,2}+\frac{7}{2}\frac{a^3}{a_{\prime}^3}b_{3,3}-\frac{9}{4}\frac{a^2}{a_{\prime}^2}b_{3,4}\Big\}\Big\}e\cos(4nt-3n_{\prime}t+4\varepsilon-3\varepsilon_{\prime}-\varpi) \qquad [10]\end{aligned}$$

$$\begin{aligned}&+\frac{n^2}{(6n-4n_{\prime})(4n-4n_{\prime})}\Big\{\frac{3(5n-4n_{\prime})^2}{2n^2}r_4-\frac{m_{\prime}}{\mu}\Big\{\frac{10n}{(5n-4n_{\prime})}\Big\{-\frac{a^2}{4a_{\prime}^2}b_{3,3}-\frac{a^3}{2a_{\prime}^3}b_{3,4}+\frac{3a^2}{4a_{\prime}^2}b_{3,5}\Big\}\\&\qquad-\frac{a^2}{a_{\prime}^2}b_{3,3}+\frac{9}{2}\frac{a^3}{a_{\prime}^3}b_{3,4}-\frac{3a^2}{a_{\prime}^2}b_{3,5}\Big\}\Big\}e\cos(5nt-4n_{\prime}t+5\varepsilon-4\varepsilon_{\prime}-\varpi) \qquad [11]\end{aligned}$$

$$\begin{aligned}&-\frac{n^2}{2n_{\prime}(2n-2n_{\prime})}\Big\{\frac{3(n-2n_{\prime})^2}{2n^2}r_2-\frac{m_{\prime}}{\mu}\Big\{\frac{2n}{(n-2n_{\prime})}\Big\{\frac{3a^2}{4a_{\prime}^2}b_{3,1}-\frac{a^3}{2a_{\prime}^3}b_{3,2}-\frac{a^2}{4a_{\prime}^2}b_{3,3}\Big\}\\&\qquad+\frac{3a^2}{2a_{\prime}^2}b_{3,1}-\frac{a^3}{a_{\prime}^3}b_{3,2}+\frac{a^2}{2a_{\prime}^2}b_{3,3}\Big\}\Big\}e\cos(nt-2n_{\prime}t+\varepsilon-2\varepsilon_{\prime}+\varpi) \qquad [12]\end{aligned}$$

$$\begin{aligned}&+\frac{n^2}{(n-3n_{\prime})(3n-3n_{\prime})}\Big\{\frac{3(2n-3n_{\prime})^2}{2n^2}r_3-\frac{m_{\prime}}{\mu}\Big\{\frac{4n}{(2n-3n_{\prime})}\Big\{\frac{3a^2}{4a_{\prime}^2}b_{3,2}-\frac{a^3}{2a_{\prime}^3}b_{3,3}-\frac{a^2}{4a_{\prime}^2}b_{3,4}\Big\}\\&\qquad+\frac{9a^2}{4a_{\prime}^2}b_{3,2}-2\frac{a^3}{a_{\prime}^3}b_{3,3}+\frac{3a^2}{4a_{\prime}^2}b_{3,4}\Big\}e\cos(2nt-3n_{\prime}t+2\varepsilon-3\varepsilon_{\prime}+\varpi) \qquad [13]\end{aligned}$$

$$\begin{aligned}&+\frac{n^2}{(2n-4n_{\prime})(4n-4n_{\prime})}\Big\{\frac{3(3n-4n_{\prime})^2}{2n^2}r_4-\frac{m_{\prime}}{\mu}\Big\{\frac{6n}{(3n-4n_{\prime})}\Big\{\frac{3a^2}{4a_{\prime}^2}b_{3,3}-\frac{a^3}{2a_{\prime}^3}b_{3,4}-\frac{a^2}{4a_{\prime}^2}b_{3,5}\Big\}\\&\qquad+3\frac{a^3}{a_{\prime}^3}b_{3,3}-3\frac{a^3}{a_{\prime}^3}b_{3,4}+\frac{a^2}{a_{\prime}^2}b_{3,5}\Big\}e\cos(3nt-4n_{\prime}t+3\varepsilon-4\varepsilon_{\prime}+\varpi) \qquad [14]\end{aligned}$$

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(n-n_{\prime})(n+n_{\prime})2a_{\prime}^2}a^2 b_{3,1}e_{\prime}\cos(n_{\prime}t+\varepsilon_{\prime}-\varpi_{\prime})$$ [15]

Expression for the reciprocal of the radius vector, when $a < a_{\prime}$.

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(n-n_{\prime})(3n-n_{\prime})}\left\{\frac{4n}{(2n-n_{\prime})}\left\{\frac{3a^2}{4a_{\prime}^2}b_{3,1}-\frac{a}{2a_{\prime}}b_{3,2}-\frac{a^2}{4a_{\prime}^2}b_{3,3}\right\}\right.$$

$$\left.-\frac{9}{4}\frac{a^2}{a_{\prime}^2}b_{3,1}+\frac{2a}{a_{\prime}}b_{3,2}-\frac{a^2}{4a_{\prime}^2}b_{3,3}\right\}e_{\prime}\cos(2nt-n_{\prime}t+2\varepsilon-\varepsilon_{\prime}-\varpi_{\prime})$$ [17]

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(2n-n_{\prime})(4n-2n_{\prime})}\left\{\frac{6n}{(3n-2n_{\prime})}\left\{\frac{3a^2}{4a_{\prime}^2}b_{3,2}-\frac{a}{2a_{\prime}}b_{3,3}-\frac{a^2}{4a_{\prime}^2}b_{3,4}\right\}\right.$$

$$\left.-\frac{3a^2}{a_{\prime}^2}b_{3,2}+\frac{3a}{a_{\prime}}b_{3,3}-\frac{a^2}{2a_{\prime}^2}b_{3,4}\right\}e_{\prime}\cos(3nt-2n_{\prime}t+3\varepsilon-2\varepsilon_{\prime}-\varpi_{\prime})$$ [18]

$$-\frac{m_{\prime}}{\mu}\frac{n^3}{(3n-2n_{\prime})(5n-3n_{\prime})}\left\{\frac{8n}{(4n-3n_{\prime})}\left\{\frac{3a^2}{4a_{\prime}^2}b_{3,3}-\frac{a}{2a_{\prime}}b_{3,4}-\frac{a^2}{4a_{\prime}^2}b_{3,5}\right\}\right.$$

$$\left.-\frac{15}{4}\frac{a^2}{a_{\prime}^2}b_{3,3}+\frac{4a}{a_{\prime}}b_{3,4}-\frac{3a^2}{4a_{\prime}^2}b_{3,5}\right\}e_{\prime}\cos(4nt-3n_{\prime}t+4\varepsilon-3\varepsilon_{\prime}-\varpi_{\prime})$$ [19]

$$+\frac{m_{\prime}}{\mu}\frac{n^2}{2n_{\prime}(2n-2n_{\prime})}\left\{\frac{2n}{(n-2n_{\prime})}\left\{\frac{2a^2}{a_{\prime}^2}-\frac{a^2}{2a_{\prime}^2}b_{3,0}-\frac{a}{2a_{\prime}}b_{3,1}+\frac{3a^2}{4a_{\prime}^2}b_{3,2}\right\}\right.$$

$$\left.+\frac{2a^2}{a_{\prime}^2}+\frac{a^2}{a_{\prime}^2}b_{3,0}-\frac{a}{a_{\prime}}b_{3,1}\right\}e_{\prime}\cos(nt-2n_{\prime}t+\varepsilon-\varepsilon_{\prime}+\varpi_{\prime})$$ [20]

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(n-3n_{\prime})(3n-3n_{\prime})}\left\{\frac{4n}{(2n-3n_{\prime})}\left\{-\frac{a^2}{4a_{\prime}^2}b_{3,1}-\frac{a}{2a_{\prime}}b_{3,2}+\frac{3a^2}{4a_{\prime}^2}b_{3,3}\right\}\right.$$

$$\left.+\frac{3a^2}{4a_{\prime}^2}b_{3,1}-\frac{2a}{a_{\prime}}b_{3,2}+\frac{3a^2}{4a_{\prime}^2}b_{3,3}\right\}e_{\prime}\cos(2nt-3n_{\prime}t+2\varepsilon-3\varepsilon_{\prime}+\varpi_{\prime})$$ [21]

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(2n-4n_{\prime})(4n-4n_{\prime})}\left\{\frac{6n}{(3n-4n_{\prime})}\left\{-\frac{a^2}{4a_{\prime}^2}b_{3,2}-\frac{a}{2a_{\prime}}b_{3,3}+\frac{3a^2}{4a_{\prime}^2}b_{3,4}\right\}\right.$$

$$\left.+\frac{a^2}{a_{\prime}^2}b_{3,2}-\frac{3a}{a_{\prime}}b_{3,3}+\frac{3a^2}{2a_{\prime}^2}b_{3,4}\right\}e_{\prime}\cos(3nt-4n_{\prime}t+3\varepsilon-4\varepsilon_{\prime}+\varpi_{\prime})$$ [22]

$$-\frac{m_{\prime}}{\mu}\frac{n^2}{(3n-5n_{\prime})(5n-5n_{\prime})}\left\{\frac{8n}{(4n-5n_{\prime})}\left\{-\frac{a^2}{4a_{\prime}^2}b_{3,3}-\frac{a}{2a_{\prime}}b_{3,4}+\frac{3a^2}{4a_{\prime}^2}b_{3,5}\right\}\right.$$

$$\left.+\frac{5}{4}\frac{a^2}{a_{\prime}^2}b_{3,3}-\frac{4a}{a_{\prime}}b_{3,4}+\frac{9}{4}\frac{a^2}{a_{\prime}^2}b_{3,5}\right\}e_{\prime}\cos(4nt-5n_{\prime}t+4\varepsilon-5\varepsilon_{\prime}+\varpi_{\prime})$$ [23]

Substituting in the equations of p. 45, for R_n their values and neglecting e^2, $e\,e_{\prime}$, $e_{\prime}^2$, and $\sin^2\frac{\iota_{\prime}}{2}$, Expression for the longitude when $a < a_{,}$.

$$\lambda = \mathrm{n}\,t + \varepsilon + \mathrm{e} \sin(\mathrm{n}\,t + \varepsilon - \varpi)$$

$$+ \frac{n}{(n - n_{,})} \left\{ 2\, r_1 - \frac{m_{,} n}{\mu\,(n - n_{,})} \left(\frac{a^2}{a_{,}^2} - \frac{a}{a_{,}} b_{1,1} \right) \right\} \sin(n\,t - n_{,}t + \varepsilon - \varepsilon_{,}) \qquad [1]$$

$$+ \frac{n}{(2\,n - 2\,n_{,})} \left\{ 2\, r_2 + \frac{m_{,} n}{\mu\,(n - n_{,})} \frac{a}{a_{,}} b_{1,2} \right\} \sin(2\,n\,t - 2\,n_{,}t + 2\,\varepsilon - 2\,\varepsilon_{,}) \qquad [2]$$

$$+ \frac{n}{(3\,n - 3\,n_{,})} \left\{ 2\, r_3 + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,3} \right\} \sin(3\,n\,t - 3\,n_{,}\,t + 3\,\varepsilon - 3\,\varepsilon_{,}) \qquad [3]$$

$$+ \frac{n}{(4\,n - 4\,n_{,})} \left\{ 2\, r_4 + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,4} \right\} \sin(4\,n\,t - 4\,n_{,}\,t + 4\,\varepsilon - 4\,\varepsilon_{,}) \qquad [4]$$

$$+ \frac{n}{(5\,n - 5\,n_{,})} \left\{ 2\, r_5 + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,5} \right\} \sin(5\,n\,t - 5\,n_{,}\,t + 5\,\varepsilon - 5\,\varepsilon_{,}) \qquad [5]$$

$$+ \frac{n}{n_{,}} \left\{ 2 \left(r_6 + \frac{r_1}{2} \right) + \frac{m_{,} n}{\mu\, n_{,}} \left(- \frac{3\,a^2}{2\,a_{,}^2} + \frac{3\,a^2}{2\,a_{,}^2} b_{3,0} - \frac{a^3}{2\,a_{,}^3} b_{3,1} - \frac{a^2}{4\,a_{,}^2} b_{3,2} \right) \right.$$

$$\left. - \frac{m_{,} n}{\mu\,(n - n_{,})} \left(\frac{a^2}{a_{,}^2} - \frac{a}{a_{,}} b_{1,1} \right) \right\} e \sin(n_{,}\,t + \varepsilon - \varpi) \qquad [6]$$

$$+ \frac{n}{(2\,n - n_{,})} \left\{ 2 \left(r_8 + \frac{r_1}{2} \right) - \frac{m_{,} n}{\mu\,(2\,n - n_{,})} \left(\frac{a^2}{2\,a_{,}^2} - \frac{a^2}{2\,a_{,}^2} b_{3,0} - \frac{a^3}{2\,a_{,}^3} b_{3,1} + \frac{3\,a^2}{4\,a_{,}^2} b_{3,2} \right) \right.$$

$$\left. - \frac{m_{,} n}{\mu\,(n - n_{,})} \left(\frac{a^2}{a_{,}^2} - \frac{a}{a_{,}} b_{1,1} \right) \right\} e \sin(2\,n\,t - n_{,}\,t + 2\,\varepsilon - \varepsilon_{,} - \varpi) \qquad [8]$$

$$+ \frac{n}{(3\,n - 2\,n_{,})} \left\{ 2 \left(r_9 + \frac{r_2}{2} \right) - \frac{2\,m_{,} n}{\mu\,(3\,n - 2\,n_{,})} \left(- \frac{a^2}{4\,a_{,}^2} b_{3,1} - \frac{a^3}{2\,a_{,}^3} b_{3,2} + \frac{3\,a^2}{4\,a_{,}^2} b_{3,3} \right) \right.$$

$$\left. + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,2} \right\} e \sin(3\,n\,t - 2\,n_{,}\,t + 3\,\varepsilon - 2\,\varepsilon_{,} - \varpi) \qquad [9]$$

$$+ \frac{n}{(4\,n - 3\,n_{,})} \left\{ 2 \left(r_{10} + \frac{r_3}{2} \right) - \frac{3\,m_{,} n}{\mu\,(4\,n - 3\,n_{,})} \left(- \frac{a^2}{4\,a_{,}^2} b_{3,2} - \frac{a^3}{2\,a_{,}^3} b_{3,3} + \frac{3\,a^2}{4\,a_{,}^2} b_{3,4} \right) \right.$$

$$\left. + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,3} \right\} e \sin(4\,n\,t - 3\,n_{,}\,t + 4\,\varepsilon - 3\,\varepsilon_{,} - \varpi) \qquad [10]$$

$$+ \frac{n}{(5\,n - 4\,n_{,})} \left\{ 2 \left(r_{11} + \frac{r_4}{2} \right) - \frac{4\,m_{,} n}{\mu\,(5\,n - 4\,n_{,})} \left(- \frac{a^2}{4\,a_{,}^2} b_{3,3} - \frac{a^3}{2\,a_{,}^3} b_{3,4} + \frac{3\,a^2}{4\,a_{,}^2} b_{3,5} \right) \right.$$

$$\left. + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,4} \right\} e \sin(5\,n\,t - 4\,n_{,}\,t + 5\,\varepsilon - 4\,\varepsilon_{,} - \varpi) \qquad [11]$$

$$+ \frac{n}{(n - 2\,n_{,})} \left\{ 2 \left(r_{12} + \frac{r_2}{2} \right) - \frac{2\,m_{,} n}{\mu\,(n - 2\,n_{,})} \left(\frac{3\,a^2}{4\,a_{,}^2} b_{3,1} - \frac{a^3}{2\,a_{,}^3} b_{3,2} - \frac{a^2}{4\,a_{,}^2} b_{3,3} \right) \right.$$

$$\left. + \frac{m_{,} n\, a}{\mu\,(n - n_{,})\, a_{,}} b_{1,2} \right\} e \sin(n\,t - 2\,n_{,}\,t + \varepsilon - 2\,\varepsilon_{,} + \varpi) \qquad [12]$$

Expression for the longitude when $a < a_{\prime}$.

$$+ \frac{n}{(3n-3n_{\prime})}\left\{2\left(r_{13}+\frac{r_3}{2}\right)-\frac{3m_{\prime}n}{\mu(2n-3n_{\prime})}\left(\frac{3a^2}{4a_{\prime}^2}b_{3,2}-\frac{a^3}{2a_{\prime}^3}b_{3,3}-\frac{a^2}{4a_{\prime}^2}b_{3,4}\right)\right.$$
$$\left.+\frac{m_{\prime}n\,a}{\mu(n-n_{\prime})a_{\prime}}b_{1,3}\right\}e\sin(2nt-3n_{\prime}t+2\varepsilon-3\varepsilon_{\prime}+\varpi) \qquad [13]$$

$$+ \frac{n}{(3n-4n_{\prime})}\left\{2\left(r_{14}+\frac{r_4}{2}\right)-\frac{4m_{\prime}n}{\mu(3n-4n_{\prime})}\left(\frac{3a^2}{4a_{\prime}^2}b_{3,3}-\frac{a^3}{2a_{\prime}^3}b_{3,4}-\frac{a^2}{4a_{\prime}^2}b_{3,5}\right)\right.$$
$$\left.+\frac{m_{\prime}n\,a}{\mu(n-n_{\prime})a_{\prime}}b_{1,4}\right\}e\sin(3nt-4n_{\prime}t+3\varepsilon-4\varepsilon_{\prime}+\varpi) \qquad [14]$$

$$+\frac{2n}{n_{\prime}}r_{15}e_{\prime}\sin(n_{\prime}t+\varepsilon_{\prime}-\varpi_{\prime}) \qquad [15]$$

$$+ \frac{n}{(2n-n_{\prime})}\left\{2r_{17}-\frac{2m_{\prime}n}{\mu(2n-n_{\prime})}\left(\frac{3a^2}{4a_{\prime}^2}b_{3,1}-\frac{a}{2a_{\prime}}b_{3,2}\right.\right.$$
$$\left.\left.-\frac{a^2}{4a_{\prime}^2}b_{3,3}\right)\right\}e_{\prime}\sin(2nt-n_{\prime}t+2\varepsilon-\varepsilon_{\prime}-\varpi_{\prime}) \qquad [17]$$

$$+ \frac{n}{(3n-2n_{\prime})}\left\{2r_{18}-\frac{3m_{\prime}n}{\mu(3n-2n_{\prime})}\left(\frac{3a^2}{4a_{\prime}^2}b_{3,2}-\frac{a}{2a_{\prime}}b_{3,3}\right.\right.$$
$$\left.\left.-\frac{a^2}{4a_{\prime}^2}b_{3,4}\right)\right\}e_{\prime}\sin(3nt-2n_{\prime}t+3\varepsilon-2\varepsilon_{\prime}-\varpi_{\prime}) \qquad [18]$$

$$+ \frac{n}{(4n-3n_{\prime})}\left\{2r_{19}-\frac{4m_{\prime}n}{\mu(4n-3n_{\prime})}\left(\frac{3a^2}{4a_{\prime}^2}b_{3,3}-\frac{a}{2a_{\prime}}b_{3,4}\right.\right.$$
$$\left.\left.-\frac{a^2}{4a_{\prime}^2}b_{3,3}\right)\right\}e_{\prime}\sin(4nt-3n_{\prime}t+4\varepsilon-3\varepsilon_{\prime}-\varpi_{\prime} \qquad [19]$$

$$+ \frac{n}{(n-2n_{\prime})}\left\{2r_{20}-\frac{m_{\prime}n}{\mu(n-2n_{\prime})}\left(\frac{2a^2}{a_{\prime}^2}-\frac{a^2}{2a_{\prime}^2}b_{3,0}-\frac{a}{2a_{\prime}}b_{3,1}\right.\right.$$
$$\left.\left.+\frac{3}{4}\frac{a^2}{a_{\prime}^2}b_{3,2}\right)\right\}e_{\prime}\sin(nt-2n_{\prime}t+\varepsilon-2\varepsilon_{\prime}+\varpi_{\prime}) \qquad [20]$$

$$+ \frac{n}{(2n-3n_{\prime})}\left\{2r_{21}-\frac{m_{\prime}n}{\mu(2n-3n_{\prime})}\left(-\frac{a^2}{4a_{\prime}^2}b_{3,1}-\frac{a}{2a_{\prime}}b_{3,2}\right.\right.$$
$$\left.\left.+\frac{3a^2}{4a_{\prime}^2}b_{3,3}\right)\right\}e_{\prime}\sin(2nt-3n_{\prime}t+2\varepsilon-3\varepsilon_{\prime}+\varpi_{\prime}) \qquad [21]$$

$$+ \frac{n}{(3n-4n_{\prime})}\left\{2r_{22}-\frac{m_{\prime}n}{\mu(3n-4n_{\prime})}\left(-\frac{a^2}{4a_{\prime}^2}b_{3,2}-\frac{a}{2a_{\prime}}b_{3,3}\right.\right.$$
$$\left.\left.+\frac{3a^2}{4a_{\prime}^2}b_{3,4}\right)\right\}e_{\prime}\sin(3nt-4n_{\prime}t+3\varepsilon-4\varepsilon_{\prime}+\varpi_{\prime}) \qquad [22]$$

The expression for the tangent of the latitude has already been given, p. 49.

When the latitude is reckoned from the plane of the orbit of the planet P, the following terms must be added, in the general case where ι is not equal to zero, to that expression;

$$
\begin{aligned}
&- \frac{n^2}{2(n-n_{\prime})(3n-n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,1} \sin(2nt - n_{\prime}t + 2\varepsilon - \varepsilon_{\prime} - \nu) \\
&+ \frac{n^2}{2(n-n_{\prime})(n+n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,1} \sin(n_{\prime}t + \varepsilon_{\prime} - \nu_{\prime}) \\
&- \frac{n^2}{2(2n-2n_{\prime})(4n-2n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,2} \sin(3nt - 2n_{\prime}t + 3\varepsilon - 2\varepsilon_{\prime} - \nu) \\
&- \frac{n^2}{4n_{\prime}(2n-2n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,2} \sin(nt - 2n_{\prime}t + \varepsilon - 2\varepsilon_{\prime} + \nu) \\
&- \frac{n^2}{2(3n-3n_{\prime})(5n-3n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,3} \sin(4nt - 3n_{\prime}t + 4\varepsilon - 3\varepsilon_{\prime} - \nu) \\
&+ \frac{n^2}{2(n-3n_{\prime})(3n-3n_{\prime})} \frac{a^2}{a_{\prime}^2} \tan \iota\, b_{3,3} \sin(2nt - 3n_{\prime}t + 2\varepsilon - 3\varepsilon_{\prime} + \nu)
\end{aligned}
$$

All the equations hitherto given apply to the case of an inferior disturbed by a superior planet, or when $a_{\prime} > a$, in order to render them applicable to the case when $a_{\prime} < a$ it is necessary to write a instead of $a_{\prime}$ in the denominator of the terms multiplied by $b_{1,n}$, and a^3 instead of $a_{\prime}^3$ in the denominator of the terms multiplied by $b_{3,n}$, in the disturbing function R, but the expressions for the quantities q are not the same in this case.

It will I think be admitted that the expressions which occur in the theory of the disturbances of the planets are more simple in terms of the quantities of which the general symbol is b, than in terms of the partial differential coefficients of the quantity called A in the notation of the Mécanique Céleste. The development of the disturbing function R in terms of the differential coefficients $\frac{dA}{da}$, $\frac{dA}{da_{\prime}}$, &c. admits of reductions, so that it may be expressed in terms of the differential coefficients of A with respect to a only. In this state it has been left by LAPLACE as may be seen, vol. ii. p. 12, but the coefficients of the terms multiplied by the squares and products of the eccentricities may be expressed very simply in terms of the quantities of which the general symbol is b, by means of reductions, of which two examples are given in the Théor. Anal. vol. i. p. 362. Similar reductions are applicable to the terms in $r\left(\frac{dR}{dr}\right)$ multiplied by the first power of the eccentricities.

In LAPLACE's notation

$$b_{1,0} = \tfrac{1}{2} b_{\frac{1}{2}}^{(0)}, \quad b_{1,1} = b_{\frac{1}{2}}^{(1)}, \quad b_{1,2} = b_{\frac{1}{2}}^{(2)}, \quad b_{1,3} = b_{\frac{1}{2}}^{(3)}$$

$$b_{3,0} = \tfrac{1}{2} b_{\frac{3}{2}}^{(0)}, \quad b_{3,1} = b_{\frac{3}{2}}^{(1)}, \quad b_{3,2} = b_{\frac{3}{2}}^{(2)}, \quad b_{3,3} = b_{\frac{3}{2}}^{(3)}$$

$$b_{5,0} = \tfrac{1}{2} b_{\frac{5}{2}}^{(0)}, \quad b_{5,0} = b_{\frac{5}{2}}^{(1)}, \quad b_{5,2} = b_{\frac{5}{2}}^{(2)}, \quad b_{5,3} = b_{\frac{5}{2}}^{(3)}$$

$$\frac{a}{a_{\prime}} b_{3,0} - \tfrac{1}{2} b_{3,1} = -\tfrac{1}{2} \frac{d b_{\frac{1}{2}}^{(0)}}{d\alpha}, \quad \frac{a}{a_{\prime}} b_{3,1} - b_{3,0} - \tfrac{1}{2} b_{3,2} = -\frac{d b_{\frac{1}{2}}^{(1)}}{d\alpha}$$

$$\frac{a}{a_{\prime}} b_{3,2} - \tfrac{1}{2} b_{3,1} - \tfrac{1}{2} b_{3,3} = -\frac{d b_{\frac{1}{2}}^{(2)}}{d\alpha}, \quad \frac{a}{a_{\prime}} b_{3,3} - \tfrac{1}{2} b_{3,2} - \tfrac{1}{2} b_{3,4} = -\frac{d b_{\frac{1}{2}}^{(3)}}{d\alpha}$$

$$3\left\{\frac{a}{a_{\prime}} b_{5,0} - \tfrac{1}{2} b_{5,1}\right\} = -\tfrac{1}{2} \frac{d b_{\frac{3}{2}}^{(0)}}{d\alpha}, \quad 3\left\{\frac{a}{a_{\prime}} b_{5,1} - b_{5,0} - \tfrac{1}{2} b_{5,2}\right\} = -\frac{d.b_{\frac{3}{2}}^{(1)}}{d\alpha}$$

$$3\left\{\frac{a}{a_{\prime}} b_{5,2} - \tfrac{1}{2} b_{5,1} - \tfrac{1}{2} b_{5,3}\right\} = -\frac{d.b_{\frac{3}{2}}^{(2)}}{d\alpha}, \quad 3\left\{\frac{a}{a_{\prime}} b_{5,3} - \tfrac{1}{2} b_{5,2} - \tfrac{1}{2} b_{5,4}\right\} = -\frac{d.b_{\frac{3}{2}}^{(3)}}{d\alpha}$$

The numerical values of these quantities are given for the principal planets in the third volume of the Mécanique Céleste.

The following numerical examples will serve to explain the expressions given above, and to show their accuracy, the results agreeing exactly with those given in the Mécanique Céleste.

$$\frac{r}{a} = 1 + \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{a^2}{2a_{\prime}^2} b_{3,1}\right\} - e\left\{1 + \frac{m_{\prime}}{\mu}\left\{\frac{{}^{*}a^3}{3a_{\prime}^3} b_{3,0} - \frac{5}{12}\frac{a^2}{a_{\prime}^2} b_{3,1}\right\}\right\}\cos(nt + \varepsilon - \varpi)$$

$$- r_1 \cos(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime})$$ See p. 55.

* The coefficient of $\cos(nt + \varepsilon - \varpi)$ p. 52 line 11, and p. 55 line 3, should be

$$-e\left\{1 + \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{3a_{\prime}^3} b_{3,0} - \frac{5}{12}\frac{a^2}{a_{\prime}^2} b_{3,1}\right\}\right\}.$$

$$r_1 = \frac{m_{,}}{\mu} \frac{n^2}{(2n - n_{,})\,n_{,}} \left\{ \frac{2n}{(n - n_{,})} \left(\frac{a^2}{a_{,}^2} - \frac{a}{a_{,}} b_{1,1} \right) + \frac{a^2}{a_{,}^2} + \frac{a^2}{a_{,}^2} \left(\frac{a}{a_{,}} b_{3,1} - b_{3,0} - \frac{1}{2} b_{3,2} \right) \right\}$$ See p. 57.

$$\lambda = n\,t + \varepsilon + \frac{n}{(n - n_{,})} \left\{ 2\,r_1 - \frac{m_{,} n}{\mu\,(n - n_{,})} \left(\frac{a^2}{a_{,}^2} - \frac{a}{a_{,}} b_{1,1} \right) \right\} \sin\,(n\,t - n_{,}t + \varepsilon - \varepsilon_{,})$$

In the theory of Jupiter disturbed by Saturn,

$\frac{a}{a_{,}} = \cdot 54531725,$ $\quad b_{1,1} = \cdot 6206406,$ $\quad b_{3,0} = \frac{4 \cdot 358387}{2} = 2 \cdot 179193$

$b_{3,1} = 3 \cdot 185493$ $\quad b_{3,2} = 2 \cdot 082131$ $\quad \frac{m_{,}}{\mu} = \frac{1}{3359 \cdot 4}$

$n = 337210 \cdot 78$ $\quad n_{,} = 135792 \cdot 34$

$a = 5 \cdot 20116636$ $\quad e = \cdot 0480767$

See Méc. Cél. vol. iii. p. 61 & 82.

Whence

$\frac{a^3}{a_{,}^3} b_{3,0} = \cdot 353381,$ $\quad \frac{a^3}{2\,a_{,}^2} b_{3,1} = \cdot 473636$ $\quad \frac{a^3}{a_{,}^3} b_{3,0} - \frac{a^2}{2\,a_{,}^2} b_{3,1} = -\cdot 120255$

log. ·120255 $= 9 \cdot 0801033$

log. 5·20116636 $= 0 \cdot 7161007$

$9 \cdot 7962040$

$\log.\frac{3\,\mu}{m_{,}}$ $= 4 \cdot 0033829$

$5 \cdot 7928211$ = log. ·0000620613 minus

Laplace has . . . . . . ·0000620566 minus

See Méc. Cél. vol. iii. p. 121, line 5.

$\log.\left\{ \frac{5}{12} \frac{a^2}{a_{,}^2} b_{3,1} - \frac{a^3}{3\,a_{,}^3} b_{3,0} \right\} = 9 \cdot 4423277$

log. e $= 8 \cdot 6819347$

log. a $= 0 \cdot 7161007$

$8 \cdot 8403631$

$\log.\frac{\mu}{m_{,}}$ $= 3 \cdot 5262617$

$5 \cdot 3141014$ = log. ·0000206111 +

Laplace has . . . . ·0000206111 the sign omitted, Méc. Cél. vol. iii. p. 122, line 28.

Calculation of r_1, see p. 57.

$\log. n_{\prime} = 5\cdot1328751$

$\log. n = 5\cdot5279013$

$\underline{9\cdot6049738} = \log. \cdot4026928$

$1 - \frac{n_{\prime}}{n} = \cdot5973072$

$\frac{a}{a_{\prime}} b_{3,1} - b_{3,0} - \frac{1}{2} b_{3,2} = -1\cdot48315$

$\log. 1\cdot48315 = \cdot1711851$

$\log. \frac{a^2}{a_{\prime}^2} = 9\cdot4732982$

$\underline{9\cdot6444333} = \log. \cdot441045$

$\log. 1\cdot5973072 = \cdot2033884$

$\log. \cdot4026928 = 9\cdot6049738$

$\log. 3359\cdot4 = 3\cdot5262617$

$\underline{3\cdot3346239}$

$r_1 = -\cdot0001301383$

$\frac{a^2}{a_{\prime}^2} - \frac{a}{a_{\prime}} b_{1,1} = -\cdot041075$

$\log. \cdot041075 = 8\cdot6135776$

$\log. \frac{n - n_{\prime}}{n} = 0\cdot7761940$

$8\cdot8373836 = \log. \cdot0687676$

2

$-\cdot1375352$

$-\cdot441045$

$-\cdot5785802$

$\frac{a^2}{a_{\prime}^2} = \cdot297371$

$\underline{-\cdot281209}$

$\log. \cdot281209 = 9\cdot4490293$

$3\cdot3346259$

$6\cdot1144054 = \log. \cdot0001301383$

$\log. a = \cdot7161007$

$\underline{6\cdot8305061} = \log. \cdot000676871 +$

LAPLACE has . . . . $\cdot000676876 +$ p. 120.

similarly $r_2 = \cdot000556924$, $r_3 = \cdot00005809$, $r_4 = \cdot000015045$, $r_5 = \cdot0000049785$

Calculation of the coefficient of $\sin (n t - n_{\prime} t + \varepsilon - \varepsilon_{\prime})$ in the value of λ. See p. 59.

$2 r_1 = -\cdot0002602766$

$\cdot0000204702$

$\log. \cdot0002398064 = 6\cdot3798601$

$\log. \frac{n - n_{\prime}}{n} = 9\cdot7761940$

$6\cdot6036661$

$5\cdot8038801$ see p. 49.

$\underline{2\cdot4075461} = \log. 255\cdot591$ minus.

$\log. \frac{n}{n - n_{\prime}} \left\{ \frac{a^2}{a_{\prime}^2} - \frac{a}{a_{\prime}} b_{1,1} \right\} = 8\cdot8373836$

$\log. 3359\cdot4 = 3\cdot5262617$

$\underline{5\cdot3111219} = \log. \cdot0000204702$

LAPLACE has . . . $255\cdot5917$ the sign omitted, p. 120.

In the notation of the Méc. Cél. vol. iii. p. 120.

$n = n^{\text{iv}}$, $n_{\prime} = n^{\text{v}}$.

The reader is requested to make the following corrections.

The expression for r^{-3} Phil. Trans. 1830, p. 345, should be

$$r^{-3} = a^{-3}\left\{1 + \frac{3}{2}e^2\left(1 + \frac{5}{4}e^2\right) + 3e\left(1 + \frac{9}{8}e^2\right)\cos(nt - \varpi)\right.$$

$$+ \frac{9}{2}e^2\left(1 + \frac{7}{9}e^2\right)\cos(2nt - 2\varpi) + \frac{53}{8}e^3\cos(3nt - 3\varpi)$$

$$+ \frac{77}{8}e^4\cos(4nt - 4\varpi)$$

The third term (multiplied by d t) in the equations of p. 23, line 4 and 6, and the second term in the equation of the same page, line 11, must be suppressed.

p. 30, line 21, read $+ m_{,}\left\{\frac{a}{a_{,}^2}\left(\cos^2\frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2}\right)\right.$ for $m_{,}\left\{-\frac{a}{a_{,}^2}\left(\cos^2\frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2}\right)\right.$

p. 32, line 5, read $+ m_{,}\left\{\frac{2a}{a_{,}^2} - \frac{a}{2a_{,}^2}b_{3,0} - \frac{a_{,}^2}{2a^3}b_{3,1}\right.$ &c. for $+ m_{,}\left\{\frac{a}{2a_{,}^2} - \frac{a}{2a_{,}^2}b_{3,0} - \frac{a_{,}^2}{2a_{,}^3}b_{3,1}\right.$ &c.

p. 39, line 9, read $-\frac{21}{8}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t - z + x)$ for $-\frac{27}{8}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t - z + x)$

p. 41, line 4, read $\frac{n_{,}^2}{n^2}$ for $\frac{n^2}{n_{,}^2}$.